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Teaching And Exploring Mathematics through the Analysis of Student's Errors in Solving Mathematical Problems

Lýdia Kontrová [a,*], Vladimír Biba [a], Daniela Šusteková [a]

[a] Department of Quantitative Methods and Economic Informatics, University of Žilina, Slovak Republic

Abstract

For decades already, the pedagogy of mathematics education has relied primarily on the role of the teacher, who demonstrates well-functioning model examples for students to motivate and encourage their tasks. This relatively routine and stereotypical procedure: interpretation of theory - examples – independent solution of assigned tasks, we decided to research in-depth within our pedagogical practice by incorporating error as a teacher's educational strategy into mathematics teaching. We believe that explaining and justifying correct and incorrect solutions to problems is more beneficial for achieving better results in mathematics education than justifying the right solutions. Such a teaching process can lead to a more informal and better understanding of mathematical concepts. In our study, we try to reveal the potential of students' incorrect solutions in conjunction with the analysis and justification of incorrect steps when reaching the final result.

We also want to point out the difference in mathematical success when error analysis is included in teaching, compared to the traditional teaching approach only in presenting the right solutions. We tested the hypothesis statistically: If we incorporate the justification and explanation of incorrect solutions of mathematical problems into the teaching process, it is possible to achieve better results in education compared to the traditional instructional teaching process only through correct examples.

Keywords: mathematical education, potential of the errors, common errors in mathematics, pedagogical experiment, t-test.

1. Introduction

The error plays an important, sometimes even essential role in the student's life and each person. We also understand it as a specific cultural and social value. Therefore, it is necessary to

* Corresponding author
E-mail addresses: lydia.kontrova@fpedas.uniza.sk (L. Kontrová)

think about, describe, and identify the place and role of error in learning theory. The emotional perception of error in the Christian tradition opposes the rational perception of error in ancient culture – here, the error is perceived as a means for a more correct, consistent and more profound knowledge of reality.

In our school, the mistake or error is often perceived as an undesirable phenomenon, as something to be avoided, as something that both the teacher and the student are afraid of. However, the error understood in this way de–motivates (deactivates). Every failure or error in the teaching process can be productive for a person; it depends on the attitude taken in this particular situation. If mathematics teaching is understood only as of the transfer of knowledge in the form of an explanation or lecture, the teacher must avoid any mistake - not sharing incorrect information (Kuřina, 2017). Any student's lack or error must be punished in such a case because he "failed to master the subject".

If we strive to teach a creative, interactive, constructive process, errors are like milestones along the way. They point in the right direction when looking for solutions and provide us with the option to find the right results. Teaching is thus realized between two poles: Error either cursed – error either praised (Kuřina, 2017). In the introduction of our paper, we discuss how different teaching theories in the past understood errors in the learning process. Above all, we were interested in accepting the error as a positive, as a "potential for the student" in the future. We analyze different approaches of the teacher to the errors done by students. We ask ourselves questions: how to change the perception of error into a source of better understanding and education, how to remove anxiety and respect from mathematics (the source of this anxiety often lies in the approach to errors by the student's teacher). In analyzing errors, we see the benefit of the student learning to argue meaningfully, construct viable arguments, and comment on the arguments of others. Students trying to justify the logical rationale will learn more than those who do not.

Literature review

In the professional literature, we find several studies on the use of error analysis in mathematics (Adamas, 2014; McLaren 2015). The study carried out for this article differs from previous studies in mathematical content, the number of teachers and students involved in the study, and online teaching.

Loibl and Rummel (Loibl, Rummel, 2014) found that secondary school students became more aware of their knowledge gaps when analyzing exercises with errors. Demonstrative comparisons of wrong-done tasks with correctly calculated tasks have filled learning gaps. Gadgil et al. (2012) conducted a study in which students who compared incorrectly solved tasks with correctly solved tasks gained a more remarkable ability to correct their errors than students who only explained the correct procedures and problem-solving. This conclusion was subsequently supported by other researchers (Durkin, Rittle-Johnson, 2012; Kawasaki, 2010; Stark et al., 2011). Each of these researchers found students at all levels of mathematics education, from elementary school to secondary school students, who learned more than students who only faced the correct solutions of the task when analyzing them and at the same time incorrect solutions to the task. This was particularly the case when the tasks with errors done were similar to the errors they made (Kawasaki, 2010; Stark et al., 2011) added that it is essential for students to be given sufficient explanation in well-designed examples before and in addition to erroneous tasks with errors. Hejný (Hejný et al., 2004) perceives error as an element of the teacher's educational strategy and emphasizes the requirement to suppress the student's unwanted fear of error, requiring the teacher not to perceive error as an undesirable phenomenon. The error detection and process to solve it is divided into six phases:

1. identification (error presence noted),
2. error localization,
3. factual analysis of the error (why the given idea is incorrect, or what is this wrong idea related to and with which other mathematical concepts it is connected),
4. elimination of the error
5. process analysis of the error (how this error occurred),
6. forming the conlusion.

Common errors in Mathematics

This section describes the errors that we have frequently seen in undergraduate mathematics, the likely errors, and their remedies. At the beginning of each semester, we notify students of these "chronically recurring" errors. Unfortunately, we must say that the situation is not improving; on the contrary, it is getting worse. In addition, the last two years, affected by the corona crisis have worsened the situation as well.

In carrying out our experiment, we, therefore, began by identifying the most frequently recurring mathematical errors of secondary school graduates, dividing them into two groups: the errors of gymnasium secondary school students and the errors of vocational secondary school students. We used as the source the test results that students got before the start of the first semester and the final reports on the results of the Matura examination in mathematics in 2018. When completing mathematical errors, we were also interested in other countries' situations and processed information from Eric Schechter's website (more than 500 teachers from different countries published their observations on errors in the subject of mathematics in school), Paul Cox's website, as well as publications by Bradis, Minkovsky and E.A. Maxwell. We divided errors made by students into several categories.

Communication errors

These negative aspects can be relatively quickly eliminated by the teacher with sufficient supervision and thus improve the quality of work. We register them in the teacher-student relationship (or vice versa, student-teacher). The teacher often perceives the student as the enemy, is not open to students' questions, and is more focused on mathematics than on the student (whether and how the student understands the explained subject matter). The hidden negative attitude of the teacher implies the fear of students, their inability to ask questions, engage in fruitful discussion, and be an active member of the teaching process. The teacher is often tempted to communicate more with gifted or active students. Nevertheless, these are exactly the slower ones in need of our help. If we focus on students' facial expressions while teaching them, it is relatively easy to grasp their understanding (or misunderstanding) – from their facial expressions.

Many problems in teaching mathematics are also related to students' poor reading comprehension skills. In Slovakia, we have registered a significant reduction in pupils' and students' level of language culture in recent years (as evidenced by several research within the OECD countries – PISA). Students often do not understand the context or do not read the tasks to the very end, or are distracted and inconsistent when reading them. At the same time, the language of mathematics uses, in addition to the general language, specific terminology, the language of formulas, algebra and requires an understanding of nonverbal expression using diagrams, graphs and figures.

It is also necessary to include in the category of communication errors related to the student's unreadable handwriting (the student understands his written text poorly or the teacher cannot guess the content of the student's work).

Algebra errors

We can conclude that we register each of the errors we mention in this paragraph at all levels of mathematics education. Many of them are caused by the usual lack of attention or poor concentration of students at work. Sometimes it would be enough to count slower, with more focus paid to the task. Many errors could be avoided in this way. In general, we could divide these errors into errors at the primary level and errors caused by a lack of more profound theoretical knowledge. We diference many types of the algebra errors:

- Bad manipulation with algebraic expressions;
- Expression extraction errors;
- Bad/lost/assumed parenthesis;
- Not comprehensible notations,
- The errors caused by improper distribution of expressions,
- The errors caused by division by zero,
- The errors caused by formal and inaccurate knowledge,
- The errors caused by akcepting non correct additive assumptions,
- The mistakes in solving quadratic equations,
- The errors caused by unwarranted generalizations – the formula or notation may work properly in one context, but some students try to apply it in the broader context, where it may not work correctly at all. Robin Chapman also calls this type of error "crass formalism".

The errors caused by reating with infinity as with a number

2. Materials and methods

Following the theoretical basis described above, we carried out our pedagogical experiment in three steps:

1. Entry diagnostic test – the aim was to find out what mathematical errors are most often made by students of the first year of technical specialization universities.
2. Experimental teaching – students of the 1st year of the faculty of of the University of Zilina Faculty of Operation and Economics of Transport and Communications within the teaching of the subject Mathematics 2 were exposed to alternative teaching. Error analysis was included in the teaching and homework of the experimental group students. Students had to look for errors in solutions, explain the errors and make appropriate means of correction. The control group students traditionally completed the teaching only on correctly solved tasks.
3. Statistical analysis of the final test results, which the experimental and control group students passed after the completion of the subject Mathematics 2. We verified the difference in students' mathematical results if error analysis was included in teaching and homework compared to the traditional learning approach using only the correct tasks.

Phase 1. Entry diagnostic test – results evaluation

The first stage of the pedagogical experiment evidenced the participation of, in total, 65 students of specialization in the transport of the first year of the of the University of Zilina Faculty of Operation and Economics of Transport and Communications, of which 37 were graduates of vocational secondary schools and 28 graduates of gymnasium secondary schools.

In the first week of the semester, students passed a diagnostic test. This consisted of 20 solved tasks of secondary school mathematics. Their task was to evaluate the correctness (incorrectness) of solving each of the twenty assigned tasks. They received 1 point for each correct statement. The maximum number of possible points was 20. The evaluation of the test reflects which errors are most common among students and which areas need to be deeper and more precisely focused on when teaching university mathematics.

We checked the test results considering two points of view. We focused on the score obtained by individual students, the average number of points in subgroups (a group of grammar secondary school students and a group of students from secondary vocational schools) and the average number of points from the test as a whole (Table 1). The maximum number of points achieved by the student was 18 and the minimum 3 points (Figure 1).

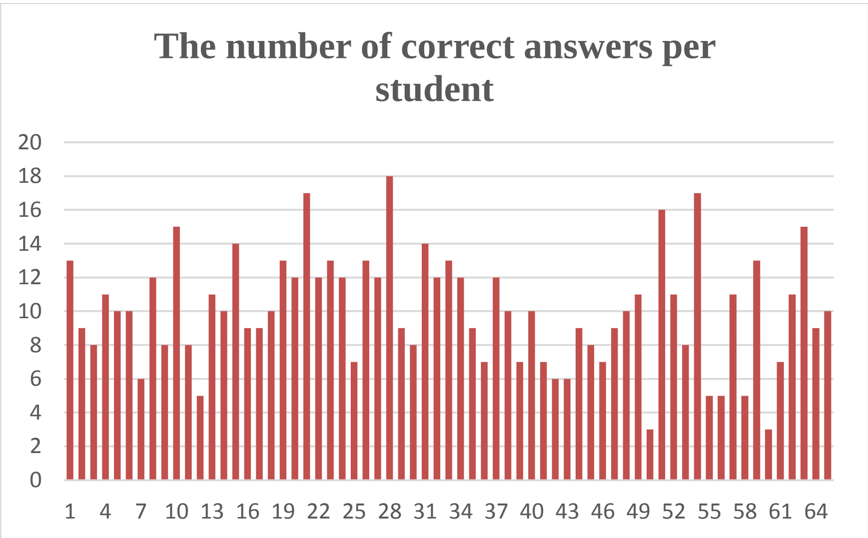

Fig. 1. Results of a diagnostic test for individual students

Table 1. Diagnostic test results by groups

Groups	Count	Average	Variance
Grammar school students	37	12,03	10,83
students of secondary vocational schools	28	8,59	11,15
all students together	65	10,31	10,99

The test results were not surprising (Figures 2, 3). Although the test contained simple tasks, which should be usual for every secondary school graduate, we observe that the average number of points obtained is at level 10 (success rate 50 %). The reasons for this situation are clear – since there is a voluntary secondary school Matura exam from the subject of Mathematics in the Slovak Republic, fewer and fewer students are opting for this "unpopular" subject. Thus, students usually experience only 3 years of mathematics in secondary school, which is insufficient, especially for those who choose universities with technical specialization.

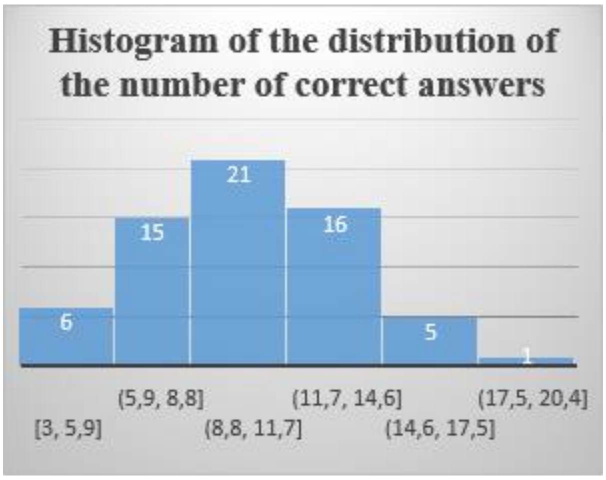

Fig. 2. Histogram of the distribution of the number of correct answers

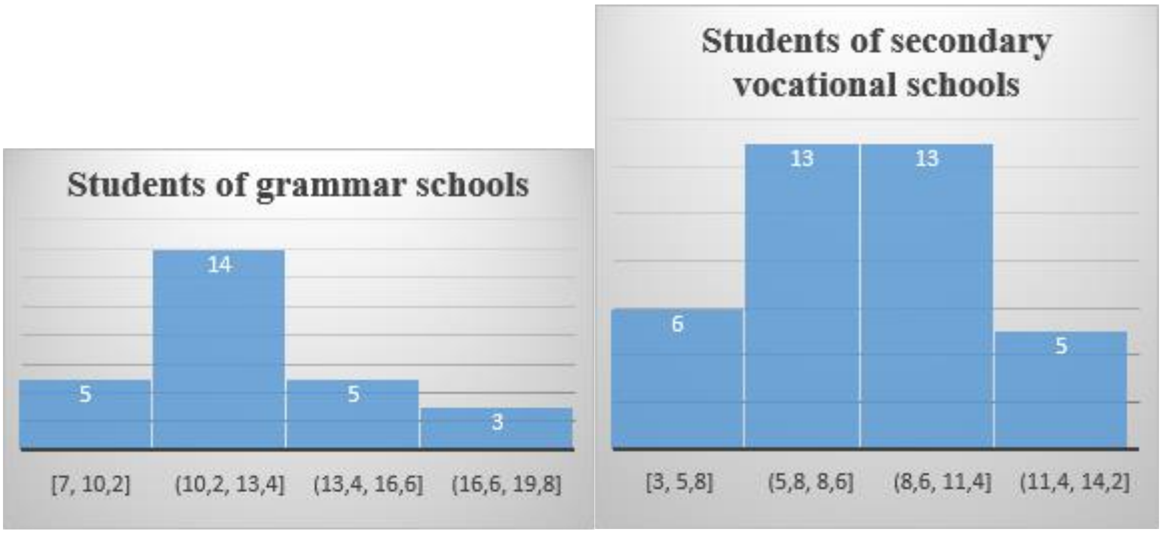

Fig. 3. Histogram of the distribution of the number of correct answers by groups

Phase 2. Experimental teaching

The information obtained from the test mentioned above was then used in the second phase of the experiment to practice examples in seminars and assign homework tasks. As part of the pedagogical experiment, we verified the effectiveness of a new way of teaching selected thematic units of the subject Mathematics 2. As mentioned above, 65 students of the first year of the University of Zilina.

Faculty of Operation and Economics of Transport and Communications participated in our experiment. The experimental group consisted of 32 students from the Air Transport Department. The teaching here was carried out experimentally. In the other groups, teaching was in a regular mode. From these groups, a control group consisting of 33 students was set up at random. The same teacher taught all groups involved in the experiment. The teaching included the analysis of incorrect solutions of assigned tasks in lectures, seminars, and homework in the experimental group. Students in the experimental group were allowed to detect errors, explain and justify errors, and discuss the correct ways to solve the assigned tasks.

Phase 3. Statistical analysis of the entrance test results

At the end of the semester, both groups were given the identical post-test from the syllabus from the subject Mathematics 2. The maximum number of points from the test was 100.

Table 2. Post-test results

	n	%	$\bar{x}$	s_x^2
Experimental group	32	73,12 %	73,12	216,9
Control group	33	65,78 %	65,78	162,9

The purpose of our study was to find out whether students of the experimental group can achieve better results from the subject Mathematics 2 if they learn using incorrectly solved assigned tasks and error analysis compared to the traditional instructional approach only with correctly solved tasks.

The following questions were answered in this study:

What was the difference in mathematical achievement when error analysis was included in students' lessons and assignments versus a traditional learning approach through correct examples only?

What kind of benefits or disadvantages did the students and teacher observe when error analysis was included in students' lessons and assignments versus a traditional learning approach through correct examples only?

Based on the formulation of the pedagogical experiment's aim, the following hypothesis was set:

H_1:The students educated with the error analysis will obtain at least an equal standard of knowledge at the end of the academic year compared to students educated without error analysis being used.

Applied tool

We analyzed the final test scores for significant differences in mean values using a two-sample parametric t–test. The observed features are X, Y, where X is the level of knowledge of students taught experimentally and Y is the level of knowledge students regularly taught. Due to the way both samples are selected, the X, Y characters are independent.

Preliminary analyses were carried out to evaluate assumptions for the t – test. Those assumptions include (a) the independence, (b) normality tested using the Shapiro–Wilk test, and (c) homogeneity of variance tested using the F test.

Methodology

To verify the hypothesis H_1, we selected a significance level $\alpha = 0{,}05$. The outcome of an experimental method we consider to be a random sample from a normal distribution $N(\mu_1, \sigma_1^2)$. The outcome of a traditional method we consider to be a random sample from a normal distribution $N(\mu_2, \sigma_2^2)$, where $\mu_1, \sigma_1^2, \mu_2, \sigma_2^2$, are unknown parameters. We had two independent files $m = 32, n = 33$.

b) We used the Shapiro-Wilka test. This test allows verifying the matching rate of the empirical probability distribution with the normal distribution. Let $(X_1, X_2, \dots, X_m)$ be a random selection from a base set with an unknown probability distribution. We will test the null hypothesis H_0, that the empirical and average probability distributions do not differ statistically and demonstrably from the alternative hypothesis that they differ.

Since p−value > α, we accepted H_0. It is assumed that the data is usually distributed. In other words, the difference between the data sample and the normal distribution is not big enough to be statistically significant.

For the character set X we get $p = 0{,}4336$; hence, if we would reject H_0, the chance of type1 error would be too high: 0.4336 (43.36%).

The larger the p−value, the more it supports H_0. For the character set Y we get $p = 0{,}1811$; hence, if we would reject H_0, the chance of type1 error would be 0.1811 (18.11 %). The assumption of a normal distribution of both samples is fulfilled.

c) We calculated the sample characteristics and by using F test, we found out that the difference between their variances is not statistically significant.

Table 3. *F* test results

F test Two−Sample for Variances		
	66	62
Mean	73,3548387 1	65,90625
Variance	229,636559 1	172,9909274
Observations	31	32
Df	30	31
F	1,32744857	
P(*F*<=*f*) one−tail	0,21858275 6	
F Critical one−tail	1,82834475 6	

The value of P (F <= f), which is stated in the row before the last one of Table 3, is the probability of error we make when we reject the tested hypothesis of equality of variances in favor of a one-sided alternative hypothesis. If this probability is less than 0,05 or 0,01, we reject the tested hypothesis at the significance level α = 0,05. Since the probability value P (F <= f) = 0,21, we cannot reject the tested hypothesis. The observed differences between the variances $\sigma_1^2,\ \sigma_2^2\ of$ samples are not statistically significant. All hypotheses for using the Student t−test were met.

We tested the difference between the two groups by a two-sample location *Student's t−test* with equal variances. We tested the hypothesis concerning the fact whether the effects of both teaching methods are the same:

H_0: $\mu_1 = \mu_2$ versus H_0: $\mu_1 \neq \mu_2$

The value of test statistics is $t = 2{,}11$ and $p = 0{,}038$ (Table 4).

When comparing it with the critical values of a *t-test,* we obtained:

$$t = 2{,}11 > t_{critical}(63) = 1{,}99.$$

H_0 hypothesis was rejected. The selective average on the selected significance level differs from the value of the average of the basic file. When using the stated teaching methods, different study results were obtained. If we apply the one-sided hypothesis

H_0: $\mu_1 = \mu_2$ versus H_0: $\mu_1 > \mu_2$

then H_0 is rejected on the significance level α if $t > t_{2\alpha}(n + m - 2)$. This was confirmed in our case as it is true that

$2{,}11 > t_{2\alpha}\ (n + m - 2) = t_{0,1}\ (63) = 1{,}67.$

The one-sided hypothesis was rejected and the difference between mean values for the stated selective file was considered statistically significant. With the help of statistical methods, it was confirmed that students educated by an innovative teaching method with the error analysis would obtain a higher standard of knowledge at the end of the academic year compared to students educated without error analysis use.

Table 4. *t*–test results

t-test: Two-Sample Assuming Equal Variances	*Variable 1*	*Variable 2*
Mean	73,125	65,78787879
Variance	223,9193548	168,0473485
Observations	32	33
Pooled Variance	195,539923	
Hypothesized Mean Difference	0	
Df	63	
t Stat	2,114867945	
P(T<=*t*) one-tail	0,019201606	
t Critical one-tail	1,669402222	
P(T<=*t*) two-tail	0,038403211	
t Critical two-tail	1,998340543	

3. Results and discussion

The main result of the described experiment was the confirmation of the hypothesis about the positive impact of an innovative teaching method with the analysis of tasks solved with errors in the teaching of the higher level of mathematics. Students who were systematically aware of the problem areas in solving examples (the most common mistakes) achieved better results in the final tests than students educated only in a traditional way by presenting the correct solutions to tasks.

The sample of students of the experimental group was from the Air Transport Department. Such sample was not random, therefore, experimental data cannot be generalized for the entire student's population.

An essential part of the implemented pedagogical experiment was the final discussion with students about their perception of the benefits (or negatives) through errors. Various open-ended questions were raised in the discussion: (a) what is your view on the use of error analysis in teaching (b) the group discussion on errors was rather constructive (productive) or confusing, (c) describe the pros or cons of using error analysis compared to by not using error analysis in the classroom teaching.

Some teachers with whom we communicated this issue had specific objections about the described teaching style. Above all, they feared the time-consuming nature of such a procedure and the possibility that students would "be confused even more" when using this way of teaching. A similar idea is shared by (Tsovaltzi et al., 2010) in their study. They concluded that exposing students to errors made could lead them later to make these errors themselves.

We were surprised by the feedback from the students that was primarily positive. The students stated that class discussions and analysis of errors in tasks and tests helped them solve their homework correctly. Two of them stated that the analysis of errors significantly helped them be better aware of their own mistakes and they enjoyed this way of teaching very much. Students also noted that error analysis has more pros than cons. In addition to the two students whose responses were unequivocally negative, another 30 students in the experimental group had positive comments on the analysis of errors. The analysis of solutions with errors provided students with the opportunity to become more involved in discussing, "explaining" and correcting the errors of the presented task and their own mistakes, which were activities that increased their interest in the learning process. The mistake acted as a specific "element of surprise" in teaching; such assignments attracted their attention, motivated them more.

4. Conclusion

The main goal of mathematics education is to support students in transitioning from their intuitive, often erroneous or incomplete knowledge to a deeper understanding of mathematical concepts. We label this informal understanding of the basic principles and interrelationships of the knowledge components as conceptual knowledge (Durkin, Rittle-Johnson, 2012). Conceptual knowledge is reflected, for example, in analytical thinking, in the ability to combine different representations of mathematical concepts, in the ability to apply mathematical knowledge in practice.

The lower tier of knowledge is formed by the procedural skills and abilities that students acquire through instructional, simple learning of standard procedures (Adams et al., 2014; Sleeman et al., 1989). Unlike procedural skills, conceptual knowledge can only be acquired through deep sensory processes. These sensory processes allow students to combine new information with previous knowledge and intuitive ideas.

To progress from procedural skills to deep conceptual understanding, students must be aware of gaps in their knowledge. Our experiment explored the possibilities of error as an element of a teacher's educational strategy. The student's error is precious information for the teacher about the level of understanding of mathematics, but above all it can be a means of finding the right way to explain concepts to students. It is a challenging but undoubtedly beneficial way (Kuřina, 2017). Implementing erroneous solutions allowed students participating in our experiment to solve a specific type of examples directly and analyze the whole context of a mathematical problem. Such practice then potentiates students to construct viable arguments, comment on their thoughts and also on the reflections of others.

Students have learned to justify a logical sequence of steps. The error analysis process has created an opportunity for them to have in-depth and meaningful discussions on alternative solutions. Learning through error analysis was enjoyable for most of the students involved. It is appropriate if the teacher is able and willing to let his students criticize and analyze the thoughts of others and make viable arguments. This is the way to reach real education. We believe that this article has opened a discussion and contributed several findings: (a) for students with significant prior knowledge of mathematics, learning incorrect solutions with errors can have positive effects on their performance and shift towards non-formal and conceptual mathematics education, (b) as we expected, the errors contained in the solutions attract the student's attention and evoke related active learning processes that activate and motivate the student, (c) if the student is unable to find the error and eliminate it, there is a problem of the lower level of the knowledge gap, such a situation needs to be diagnosed and reeducation started, (d) mistakes become a productive element of learning, especially for students who do not have profound knowledge gaps from previous mathematics education.

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