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Heuristic Potential of the History of Mathematics (on the Example of the Composition of Mathematical Problems for Schoolchildren)

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Abstract

The work is devoted to the identification of new possible ways of historicizing mathematical education, which is considered as an important pedagogical phenomenon. The purpose of the study was to reveal the heuristic potential of the historicization of mathematical education by reconstructing the substantive component of the methodological system. The study confirmed the positive impact of historical information on improving the quality of mathematical education. The conclusions were obtained on the basis of an empirical study conducted, the respondents were students, secondary school teachers and teachers of higher schools, standardized scales served as diagnostic materials. Based on the SWOT analysis, a matrix was built in which the strengths, opportunities, weaknesses and threats of the historical and methodological approach being implemented were recorded. It is proposed to use mathematical problems with a historical plot as a means of historicization. A new task bank has been developed for unified subjects “Biography and scientific heritage of L. Euler”, “Biography and scientific legacy of G.V. Leibniz”. The choice of the scientific heritage of scientists is determined by a number of circumstances. Firstly, the versatility of their scientific interests – scientists have made discoveries in various fields of mathematics, physics, and philosophy. Secondly, scientists have left a rich scientific legacy, which has not yet been fully studied and understood, and new facts about the discoveries are constantly emerging. Thirdly, the scientific results obtained by scientists are included in the content of the school curriculum.

Keywords: historicization of mathematics teaching, composition of problems with historical plots, methodological heritage.

1. Introduction

The inclusion of historical information in mathematical education makes it possible to revitalize the teaching of mathematics, make it interesting, as well as expand the scientific and

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cultural horizons of students. Obviously, the use of historical information in teaching requires additional scientific training in the field of the history of science from a mathematics teacher.

The German mathematician Felix Christian Klein (1849–1925) was one of the first to introduce historical information during lectures (Klein, 1926). In Russia, the history of mathematics was first studied at the Imperial Moscow University, thanks to the efforts of Professor V.V. Bonynin (1849–1919).

In 1959, K.A. Rybnikov (1913–2004) founded the History and Methodology of Mathematics and Mechanics Department at Lomonosov Moscow State University, whose staff developed a program for teaching the history of mathematics (list and sequence of sections), which became widespread both in classical universities and pedagogical universities (Rybnikov, 1994). Yushkevich compiled a monumental work on the history of mathematics (Yushkevich, 1992). S.S. Demidov and other students of K.A. Rybnikova continues her research activities at the present time (Demidov et al., 2018), clarifying and discovering new facts in the field of the history of mathematics, which indicates the need for constant updating of the content of the history of mathematics course (Demidov, 2021).

A modern researcher T.S. Polyakova proposed to enrich the history of mathematics course for teachers with information on the history of mathematical education (Polyakova, 2002).

Currently, the search for ways to implement historicism in teaching mathematics continues. K. Clark, T. Kjeldsen, S. Schorcht, C. Tzanakis, X. Wang propose to improve mathematical education through the introduction of historical information into the course content (Clark et al., 2016), I.A. Mendes believes that the school mathematics course should be permeated with historical information (Mendes, 2020). E. Barbin determined the volume of historical information in mathematical education (Barbin, 2022).

Researchers O.A. Pavlova, Z.F. Zaripova, L.R. Zagitova, V.G. Zakirova suggest using cases from the lives of mathematicians as a means of implementing historicism (Pavlova et al., 2021). E. Gerasimova et al. They describe the experience of introducing information on the history of mathematics for the design of programs for spiritual and moral education (Gerasimova et al., 2021). I.M. Izmirli draws attention to the fact that the issue of lack of time cannot be considered as an argument when historical information is included in the content of mathematical education (Izmirli, 2020).

Thus, the existing work on the historicization of mathematical education focuses mainly on the design of the content, rather than the means of teaching. The possibilities of using mathematical problems as a means of implementing the principle of historicism have not been studied at all.

Thus, the purpose of the study was to reveal the heuristic potential of the historicization of mathematical education by reconstructing the meaningful component of the methodological system – the design of a system of tasks.

2. Materials and methods

The research was conducted at the level of theoretical and empirical understanding.

At the first stage of the theoretical research, the problem material in mathematics textbooks was analyzed (modern school textbooks on algebra by the author A.G. Mordkovich, the author's team led by S.M. Nikolsky, the author's team led by Yu.M. Kolyagin, etc.). In order to study the content of the problem material in mathematics textbooks, the content analysis approach proposed by F. Zeynivandnezhad (Zeynivandnezhad et al., 2024) was used. It was revealed that the plots of mathematical problems do not sufficiently take into account (cultural sensitivity and inclusivity), which, in the fair opinion of the scientist, is one of the criteria for the quality of mathematical education.

A survey was also conducted, which made it possible to identify the value attitude towards the implementation of the principle of historicism in teaching mathematics and possible obstacles to its implementation. The target audience consisted of high school teachers, secondary school teachers and students enrolled in various educational programs. The representative sample was formed by stratified selection and included 200 respondents. The main characteristics of the sample are presented in Table 1. The study participants answered a questionnaire consisting of a section of bio-data (gender, age, status and educational programs) and a validated scale evaluating methodological aspects of historical material related to the value attitude towards the introduction of historicism in mathematical education. For example: "Do you think it is necessary to include information on the history of mathematics in the learning process?", "Will tasks on the history of mathematics enliven the learning process?", "What causes the difficulties of including elements of historicism in teaching mathematics?", "Is the lack of thematic methodological developments on the history of mathematics an obstacle to inclusion elements of the history of mathematics in the lesson?", "Is lack of time an obstacle to including elements of the history of mathematics in the lesson?", "Will a collection of problems on the history of mathematics be in demand in a modern school?" etc. The questionnaire included 12 questions in total. All responses were evaluated on a nominative scale. The survey process was organized through correspondence between respondents and researchers. Empirical data processing was carried out using technical tools (MS Excel) and methods of mathematical statistics (Anova). Additionally, a survey was conducted with the respondents, in which the strengths, opportunities, weaknesses and threats (SWOT analysis) of the methodological project were recorded.

Table 1. Characteristics of respondents

Variable		**Frequency**	**Frequency**
Gender	Male	72	36
	female	128	64
Age	18-24	83	41,5
	25-35	65	32,5
	>35	52	26
Status	Professor	49	24,5
	High school teacher	68	34
	Student	83	41,5
Educational programs	Education and pedagogical sciences	32	16
	Engineering, technology and technical sciences	28	14
	Mathematical and natural sciences	23	11,5

At the second stage, sources on the history of mathematics were collected, systematized and analyzed, as well as their comparison in order to identify the most reliable events and facts. At this stage, the method of historical reconstruction of events related to the life and scientific work of scientists L. Euler (1707–1783) and G. Leibniz (1646–1716) was used. The works of L. Euler and G.Leibniz and the literature on their contribution to science were analyzed. In addition to the well-known works of Thiele Rüdiger on the biography of Euler (Rüdiger, 2005) and Eric Aiton on the biography of Leibniz (Aiton, 1985), the works of modern scientists were analyzed.

New details about Leibniz's scientific activities at the Academy of Sciences are given in the article by M. Palomo (Palomo, 2021). Researcher L. Strickland has discovered new information about Leibniz's discovery of formulas for numerical series and the scientist's difficult attempts to publish this result (Strickland, 2023). Katz Mikhail G., Kuhlemann Karl, Sherry David, Ugagli Monica, based on Leibniz's letter addressed to Masson, provided a clarification in Leibniz's understanding of infinitesimals (Katz et al., 2021). Then this idea was developed from the point of view of philosophy in other works by M. Katz (Katz et al., 2021, Katz et al., 2023). Moreover, between the teams of authors Richard Arthur and David Rabouin (Arthur, Rabouin, 2024) and J. Bair, A. Borovik, V. Kanovei, M. Katz, S. Kutateladze, S. Sanders et al. (Bair et al., 2022; Bair et al., 2023), the controversy about the extent to which Leibniz's understanding of the infinitesimal differed from previous authors. O. Esquisabel, F. Raffo Quintan clarified the philosophical component in Leibniz's mathematical discoveries (Esquisabel et al., 2021). M. Rajab Nezhadian, A. Hosseini showed how the theodicy of God proposed by Leibniz can be applied to explain the philosophical categories of good and evil in modern research (Nezhadian and Eskandian, 2021).

Another famous scientist, Leonard Euler, left a huge and diverse scientific legacy. Researchers from different countries are working on understanding his works, some of which have not yet been studied and published. In a recent work by G.I. Sinkevich, the history of L. Euler's discovery of the most beautiful formula in mathematics was clarified. Another famous scientist, Leonard Euler, left a huge and versatile scientific legacy. Researchers from different countries are working on understanding his works, some of which have not yet been studied and published. In a recent work by G.I. Sinkevich, the history of the discovery of L. Euler, the most beautiful formula in mathematics, left a huge and diverse scientific legacy by another famous scientist, Leonard Euler. Researchers from different countries are working on understanding his works, some of which have not yet been studied and published. In a recent work by G.I. Sinkevich, the history of L. Euler's discovery of the most beautiful formula in mathematics was clarified. Another famous scientist, Leonard Euler, left a huge and versatile scientific legacy. Researchers from different countries are working on understanding his works, some of which have not yet been studied and published. In a recent work by G.I. Sinkevich clarified the history of L. Euler's discovery of the most beautiful formula in mathematics ($e^{i\psi} = -1$) (Sinkevich, 2023), in the study of T.A. Lavrinenko and A.A. Belyaev, L. Euler's role in the development of methods for solving diophantine equations was rethought (Lavrinenko, 2021).

G. Ferraro revealed Euler's contribution to the restructuring of mathematical analysis. He wrote: «Euler changed the architecture of this science and regarded analysis as lying at the heart of mathematics, while geometry and mechanics were considered as a sort of applied analysis» (Ferraro, 2020).

S.R. Bistafa (Bistafa, 2022) and independently of him a large team of scientists M. Fahad, S. Ali, M. Khan, M. Husnain, Z. Shafi, A. Samad and N. Jan (Fahad et al., 2021) present new arguments about the priority of L. Euler in the discovery of facts in the field of differential equations. The work of D. Bruno shows that the role of Euler's discoveries in the field of non-standard analysis has not lost its significance today (Bruno, 2023).

At the third stage, the method of modeling problems with historical content was used. Y.M. Kolyagin suggested using mathematical problems as a means of teaching and developing students. He identified three main elements in the task structure (task condition, task object and task goal), which we used for task composition (Kolyagin, 1977). V.V. Guzeev formulated the requirements for the task system structure: completeness, availability of key tasks, connectivity, increasing difficulty, target orientation and sufficiency, psychological comfort (Guzeev, 2001).

V.N. Sadovsky emphasizes the need to comply with certain principles of designing a system of tasks: integrity, structurality, purposefulness, integrativity, hierarchy (Sadovsky, 1974).

3. Results

The effectiveness of including historical information in teaching mathematics has been proven not only on a theoretical but also on a practical level. The survey showed that 97 % of respondents recognize the usefulness of including historical information in a mathematics course (the distribution by status is as follows: 50 % are students; 25 % are teachers; 25 % are higher school teachers). The majority of respondents (96.3 %) agreed that problems on the history of mathematics would introduce an element of novelty and interest into the learning process (age distribution: 18-24–52 %; 25-35–37 %; >35–11 %); 92.6 % of respondents agreed on the need to create a modern collection of tasks with a historical plot and its relevance (distribution by status: 14 % are students; 65 % are teachers; 21 % are high school teachers).

The measure of internal consistency of the test (consistency, reliability), based on the average interpoint correlation, was determined using the Cronbach's alpha analysis method in a complete sample (12 respondents) for all 12 test tasks. This number of questions in the questionnaire is optimal so that there is no artificial overestimation of the value of the Kronbach coefficient (the "weak point of the indicator"). The assumptions of this method were as follows: firstly, the testing performed is based on independent observations. Secondly, the determination of the type of distribution for each group of points on the scale by comparing the main characteristics of the sample – mode (M_o), median (M_e) and mean values (μ), revealed its normal distribution (Table 2).

Table 2. Checking for the normality of the distribution

B202 | =СРЗНАЧ(B2:B201)

	A	B	C	D	E	F	G	H	I	J	K	L	M
202	µ	1	1	2	3	1	2	2	2	1	3	2	2
203	Mo	1	1	2	3	1	2	2	3	1	3	2	2
204	Me	1	1	2	3	1	2	2	3	1	3	2	2

The obtained value of the alpha-Kronbach coefficient using the MS Excel add-in "Analysis Package" turned out to be 0.75, which indicates a good level of agreement between the positions of the scale (Table 3).

Table 3. Results of the analysis of variance

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Strings	23,71833	199	0,119188	0,410357	1	1,179863273
Columns	577,875	11	52,53409	180,8723	1,5687E-297	1,793013411
Error	635,7917	2189	0,472648			
Total	1237,385	2399				
Cronbachs Alpha	0,74783	=1-(U222/U224)				

In addition, an analysis of strengths and weaknesses, opportunities and threats (SWOT analysis) was applied. The opinion of teachers, students, and teachers of higher education was taken into account. A summary of the results is shown below.

Strengths: actualization of the values of education through immersion in the scientific atmosphere of the past; the interest of the teaching staff; consideration of the mathematical concept, method from the point of view of its historical development contributes to building the logic of the subject, more solidly assimilating it; tasks with a historical plot introduce an element of novelty and interest in the learning process; formation of students' understanding of the general methods of historical-scientific knowledge; improving the quality of knowledge (consistency, depth, strength); formation of a scientific worldview; ensuring the humanitarization of mathematical education; implementation of the educational orientation of education (development of spiritual and moral qualities of a person).

Weaknesses: lack of didactic support for the history of mathematics; excessive increase in the burden on teachers to search and comprehend relevant sources; lack of experience in training teaching staff to work with authentic mathematical texts; lack of time in mathematics lessons; poor consistency with modern curricula.

Possibilities: systematization and structuring of historical material for more effective solution of educational and developmental tasks with the possibility of implementing this approach at other levels and subjects; integration of new practice of organizing classes in the history of mathematical science into the traditional process; use of existing modern technological tools to update textbooks with historical content.

Risks: diverting students' attention from a mathematical fact to a historical one; reducing teacher participation due to an increase in their workload.

A specially designed system of mathematical problems should be considered as a means of implementing historicism.

Based on the principles clarified by us, proposed by Yu.M. Kalyagin, V.V. Guzeev and V.N. Sadovsky, an algorithm for composing problems on the history of mathematics has been developed. At the first stage, the content of the program, textbooks and problem books in mathematics is analyzed (1). At the second stage, mathematical knowledge is highlighted, which students must master in the process of solving problems (2). At the third stage, a historical topic is selected and material is selected for the formulation of the problem plot (3). At the fourth stage, tasks are formulated (4). At the fifth stage, the principle of the hierarchy of tasks is chosen (according to the sequence of historical events or according to the increasing difficulty) (5).

The most difficult stage in the composition of mathematical problems is the fourth stage (formulation of the problem plot). Its implementation can be carried out in two approaches. The first approach to the formulation of the problem's plot assumes that the result of solving the problem will be a number with which a historical event is associated. In the course of solving such a problem, the student not only consolidates mathematical knowledge, but also discovers a new historical fact for himself. For example, a possible formulation of the problem looks like this: "By simplifying the expression $3(12^2+15^2 + 2*10^2)$ find the date of birth of the famous Swiss mathematician. Tell me the name of this mathematician" (The answer is 1707 – the year of L. Euler's birth).

The second approach to the formulation of the task's plot assumes that historical facts are used in the task's plot itself. For example, linear equations are used as models for composing a problem with a historical plot on the topic "Scientific creativity of L. Euler". Task: "The pen of Leonard Euler owns 850 works, including monographs, articles in the journal Commentarii Academiae scientiarum Imperialis Metropolitanae and 430 other works of various kinds. How many fundamental monographs have been published by L. Euler, if there were 20 times fewer of them than the articles in the journal Commentarii Academiae scientiarum Imperialis Metropolitanae.

The solution of this problem leads to the formulation of the equation $21x+430=850$.

The second method of construction is more complex, since it involves in-depth study and careful selection of historical information. On the one hand, the plot of the problem should contain a real historical fact, on the other hand, the text of the problem should not be overloaded with unnecessary details, not distract from the mathematical meaning (object (according to Y.M. Kolyagin). However, this method of constructing problems is preferable, since historical facts are not artificially tied here, but they naturally introduce the history of mathematics. A set of tasks is of particular cultural value if their plots reveal the details of one big historical theme.

Thus, the following requirements for the design of tasks of different types differ.

1. Requirements for the composition of tasks that contain a question that implements a cognitive goal in the field of the history of mathematics:

1) The presence of a single mathematical/historical topic;

2) Historical problems act as a supplement that supports interest in the history of mathematics;

3) The answer to the problem should be expressed as a number.

2. Requirements for the composition of problems with historical content in the condition, i.e. historical facts from the development of mathematics are used in the plot of the problem:

1) Tasks are proposed at the stage of fixing the material;

2) The presence of a single mathematical/historical topic;

3) The task condition should not be artificially overloaded with historical facts.

To compile such tasks, a thorough study of one of the historical topics will be required. Let's take two historical themes as an example: "Biography and scientific heritage of L. Euler" and "Biography and scientific heritage of G.V. Leibniz". The choice of the scientific heritage of these scientists is determined by a number of circumstances. Firstly, the versatility of their scientific interests – they made discoveries in various fields of mathematics, physics, and philosophy. Secondly, they left a rich scientific legacy. The legacy of scientists has not yet been fully studied and comprehended.

Mathematical topic "Linear equations", historical topic: "The great German mathematician G.V. Leibniz"

1. Having solved the equation, you will find the date of birth of the famous German mathematician. Who is this scientist?

$2x+170=3462$ (The answer. 1646. This is the year of G.V. Leibniz's birth).

2. By solving the equation, you will find the date of admission of G.V. Leibniz to the University of Leipzig.

$3x+ 240= 5223$ (The answer. 1661).

3. Having solved the equation, you will find the date of foundation of one of the very first mathematical journals "Acta Eruditorum".

$2x- 310=3054$ (The answer. 1682. The journal "Acta Eruditorum" was founded by G.V. Leibniz).

4. Having solved the equation, you will find the date of G.V. Leibniz's acquaintance with Emperor Peter I.

$4x + 180 = 6968$ (The answer. 1697).

We will also give some examples of problems on the mathematical topic "Degree with a natural indicator" and the historical topic "Biography and scientific heritage of L. Euler".

1. Simplifying the expression, find the date of the opening of a scientific institution opened in St. Petersburg and associated with the activities of L. Euler.

$3879*2^4:3^2$ (The answer. 1724).

2. Simplifying the expression, find the age of L. Euler, at which he moved to Russia.

$1900: 2^2 : 5^2$ (The answer. 19 years).

4. Discussion

The historicization of mathematical education involves a revision of the content of the assessment of textbooks in mathematics. We agree with O. Sahin and M. Basgul that an important parameter for evaluating the content of textbooks should be high moral values, which should be included in the content (Sahin, Basgul, 2018). However, we consider this list to be insufficient, since it lacks the requirement of historicism.

Zeynivandnezhad identified six main parameters for the systematic evaluation of mathematics textbooks: (1) language and communication; (2) content analysis; (3) pedagogical approach; (4) cultural sensitivity and inclusivity; (5) assessment and exercises; (6) visual aids and presentations. The principle of historicization may latently be present as part of the principle of "cultural sensitivity and inclusivity" (4) proposed by F. Zeynivandnezhad et al. (Zeynivandnezhad et al., 2024).

We agree with the possibility of implementing the principle of historicism through the content of mathematical education, considered by O.A. Pavlova, Z.F. Zaripova, L.R. Zagitova, V.G. Zakirova (Pavlova et al., 2021), but we believe that this is not enough. However, the authors propose a new way to implement this principle – through the composition of mathematical problems with historical content.

The requirements for the task system considered by V.V. Guzeev and V.N. Sadovsky take into account only the mathematical component in the content of the tasks, but do not take into account the historical and cultural function of the condition, therefore they require clarification.

The approach proposed by the authors has a number of advantages over previously developed cottage materials of mathematics textbooks. The use of historical information makes it possible to solve a system of pedagogical tasks in a complex, expand scientific horizons, develop cognitive abilities, and increase interest.

4. Discussion

The introduction of historical information into the teaching of mathematics is a necessary element, since it allows you to make teaching mathematics meaningful and interesting. This fact has been confirmed by theoretical, practical and empirical studies.

The results of the study show that the design and implementation of tasks with a historical plot will ensure the actualization of educational values through immersion in the scientific atmosphere of the past years, will contribute to building the logic of the subject, more solid assimilation; introduce an element of novelty and interest into the learning process; form students' understanding of the general methods of historical and scientific knowledge, expand scientific horizons, and they also improve the quality of mathematical knowledge (consistency, depth, strength). Unlike the theoretical method of introducing historical (short stories), the proposed method of composing tasks based on a historical approach does not require additional time in the classroom. The problems that exist in mathematics textbooks today can be painlessly replaced with tasks with historical content.

The main conclusion of the study is that the success of the modernization of the content of mathematical education will be guaranteed when the proposed innovations are based to the maximum extent on the heuristic potential of historicization.

The conducted research has certain limitations. First, the content of the tasks should be coordinated with the corresponding program of the mathematics course. Moreover, the teacher should focus the students' attention not on the historical fact, but on the mathematical one, so the plot of the tasks should not be excessively voluminous. Mathematical knowledge should remain dominant in the context of problems with historical plots, and historical information should play a secondary role. Secondly, the criteria for selecting the experimental sample were such indicators as age, gender, status and types of educational programs. However, additional variables that could potentially affect the results of the experiment, such as socio-economic status or cultural differences, were not taken into account. Studies by some scientists (Muijs et al., 2009; Pribesh et al., 2011) established the relationship of individual socio-economic characteristics of a person with different levels of cognitive skills development, the pace of learning, the availability of library and information resources for the implementation of this historical and methodological approach. Moreover, cultural differences can influence the formation of spiritual and moral qualities, values, beliefs and behavior of people (Sahin, Basgul, 2018).

Further promising areas include the possibility of applying the idea and algorithm of problem composition to other topics and sections of school mathematics. An example is the biography of I. Newton.

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