European Journal of Contemporary Education
E-ISSN 2305-6746
2025. 14(1): 79-92
DOI: 10.13187/ejced.2025.1.79
https://ejce.cherkasgu.press

Characteristics of Students' Perceived Goal of Mathematics Teaching and Motivation to Learn Mathematics

Dušan Vallo [a], Tomáš Lengyelfalusy [b], Dalibor Gonda [c, *], Katarína Kútiková [b]

[a] Department of Mathematics, Faculty of Natural Sciences and Informatics, Constantine the Philosopher University in Nitra, Nitra, Slovakia
[b] Department of School Didactics, DTI University, Dubnica nad Váhom, Slovakia
[c] Department of Mathematical Methods and Operations Research, Faculty of Management Science and Informatics, University of Žilina, Žilina, Slovakia

Abstract

Currently, there is a decline in students' interest in learning mathematics. At the same time, PISA tests detect a declining level of mathematical literacy. The aim of the research described below was to find out what effect the students' perceived goal of teaching mathematics has on their motivation to learn mathematics. 239 students aged 18 and 19 participated in the research. We used a mixed methods approach to obtain the necessary data. According to our findings, students find mathematics useful but not of personal value to them. The goal of teaching mathematics, as perceived by students, is not sufficiently motivating for them to achieve good performance in learning mathematics. At the same time, they do not consider it useful for them to acquire additional new knowledge from mathematics.

Keywords: the goal of teaching mathematics, motivation, personal value, mathematics, good performance.

1. Introduction

Mathematics significantly affects a person's personal and working life and is essential for an individual's individual and working life (Maass et al., 2019). It is not surprising that several studies have shown a positive correlation between the level of mathematical skills of a school graduate and their applicability on the labour market (Rønning, 2022; Pepin et al., 2021; van der Wal et al., 2017; Bakker, Akkerman, 2014; Hoyles et al., 2010). A positive correlation was even found between mathematical education and the rate of increase in the employee's salary (Rosse, Betts, 2004). Currently, the demand of employers that school graduates have sufficiently developed

* Corresponding author
E-mail addresses: dalibor.gonda@fri.uniza.sk (D. Gonda)

mathematical skills is becoming stronger, because they support many other skills needed to solve problems professionally, but also in a person's everyday life (Gainsburg, 2015). The fact that mathematics education supports students' progress in other subjects has been one of the reasons why mathematics is considered part of a person's general education (Henn, Kaiser, 2001).

In recent years, international student testing has also shifted from testing mathematical knowledge to testing mathematical literacy. The essence of mathematical literacy is the student's ability to realize the role of mathematics in the real world and, based on it, to correctly assess and make decisions as required of a constructive, engaged and thinking citizen (Niss, 2015). Despite the stated findings about the place of mathematics in general education and its impact on applicability in the labor market, the level of students' mathematical literacy is decreasing (Foster, Schleicher, 2022). This decline in the level of acquisition of much-needed mathematical skills by students leads experts to constantly search for appropriate means to reverse this undesirable trend. The analysis of lessons within the framework of TIMSS 1999 found that the content of teaching mathematics is mainly the transfer of finished products and the learning of ways of thinking when solving tasks is absent (Givvin et al., 2009).

Students take hundreds or even thousands of hours of mathematics at school and learn various calculation algorithms and definitions that they will almost never use in everyday life or work (Boaler, 2015). According to the findings (Di Martino, Zan, 2011), this form of teaching mathematics creates a distorted image of mathematics in students, which is one of the factors creating a negative emotional disposition of the student towards mathematics and a belief about its uselessness for everyday life, thus reducing their motivation to learn with mathematics. According to 2015 Fields Medal winner Maryam Mirzakani, school math has moved away from real math. The gap that has arisen between school mathematics and real mathematics is probably the core of the problem of mathematics education. Instead of memorizing definitions and calculation procedures, he recommends that teaching mathematics focus on creatively finding own solution procedures based on already acquired knowledge. We believe that learning mathematical procedures and definitions is not the full use of the potential of mathematics as a teaching subject. Students often do not see the transfer of mathematical skills to other scientific disciplines, and even to their future professional life, i.e. j. the need to study mathematics as an essential element of a person's general education is hidden from them. The goal of our research was to find out what goals of mathematics education would be meaningful for students and, above all, motivating to learn mathematics.

2. Theoretical basis

The term "motivation" is generally seen as the energy that drives people to achieve a goal. Motivation significantly affects not only a person's decision to do something, but also how long and how intensively he will devote himself to the given activity (Han, Yin, 2016). Motivation plays one of the most important roles in achieving students' goals in a given subject (e.g. Hung et al., 2019; Brooker et al., 2018; Xiong et al., 2015). According to the source, motivation is most often divided into internal and external. Internal motivation comes from the individual himself, who is motivated by the achievement of a set goal. According to El-Adl & Alkharusi (2020), students are intrinsically motivated by goals the achievement of which has personal value for them and recognize the potential benefits they will gain from achieving this goal. Extrinsic motivation is stimulated by the environment in which the individual is (Weber, 2003) and the motivating factor can be benefits other than the achievement of a set goal. In the school environment, this benefit for the student is, for example, achieving a good evaluation. It follows from the above that the motivation itself is driven by the goal. The term "goal" means an ideal idea of what should be achieved in a given activity. According to Goal-setting theory (Locke, Latham, 2002), set goals affect individual performance through several mechanisms. Goals primarily direct an individual's attention and efforts to goal-directed activities and at the same time eliminate activities that are not relevant to goal achievement. A correctly defined goal helps to focus attention and energy primarily on those activities that are related to the set goal. Since achieving a goal requires energy, the amount of which is positively correlated with the level of difficulty of the goal (Latham, Locke, 2013), the second important function of goals is to mobilize the energy needed to achieve the goal. Another key factor in achieving a goal is persistence. The level of difficulty of the set goal mobilizes not only the amount of energy, but also its equal expenditure, so that it is enough for the individual until the moment of reaching the goal. For some goals, it is advisable to work intensively in a shorter time, for other goals, more gradually over a longer time interval. A goal set in this way encourages persistence in achieving it. Goals also influence an individual's performance indirectly by leading to the discovery and use of task-relevant knowledge and strategies (Wood, Locke, 1990). From the above overview of the basic knowledge of Goal-setting theory, it follows that the goal of an activity has a motivational potential, because it can mobilize, concentrate, and manage the internal energy necessary for its achievement in an individual.

Each subject, including mathematics, was included in general education by experts with the aim of developing the student's personality. Experts have set educational goals that students should achieve in individual subjects. However, according to Achievement goal theory, the same goals motivate students to different learning activities that are related to different goal orientations. Initially, two basic types of goal orientations were identified, which reflect different priorities of the student in learning. Mastery goals represent the target orientations of a student who wants to not only develop already acquired skills and competences in a given subject, but also acquire new skills and competences and understand new ideas. On the other hand, performance goals are a manifestation of an effort to demonstrate individual skills and abilities that the student has already acquired in the given subject. Two more were later added to the original two target orientations. Mastery-avoidance goal orientation, which represents the student's effort to retain already acquired knowledge and skills at the highest possible level.

A student set up in this way is strongly motivated by goals that encourage repetition and practice of the subject matter. Performance-avoidance goal orientation, which represents an effort to maintain a good image of one's knowledge and skills (Elliot, 1999; Elliot, Muryama, 2008). Different goal orientations of students indicate their different expectations from achieving the goal, i.e. j. for them, goals represent certain general values. They achieve these goals in such a way that the achieved goal represents a subjective value for them. Subjective values can go beyond the context of the goal itself. Therefore, we think that before a student decides to achieve a set goal, a process of evaluating the subjective importance of this goal takes place inside him – internal motivation and at the same time an estimate of the necessary "quality" of its achievement – external motivation. These two factors together determine the amount of energy available to the student to achieve the goal.

One of the most influential theories for explaining decisions in achieving study goals is the Erwartungs-Wert-Theorie (Eccles et al., 1983; Eccles, Wigfield, 2020). According to this theory, the expectation of success and the subjective value that the student perceives in relation to the set goal are central determinants of learning and performance behaviour. Even these two elements significantly influence the student's relationship to the given teaching subject. Behind the expectation of success lies the student's belief in how well he can fulfill the set goal (Eccles, Wigfield, 2020). Self-efficacy is also strongly connected with the expectation of success, especially at the global level and not in relation to specific goals (Marsh et al., 2019). Subjective value includes four sub-dimensions: the joy associated with achieving a goal, the importance of good performance to oneself, the utility of achieving a goal to oneself, and the costs necessary to achieve a goal (Eccles 2005; Eccles, Wigfield, 2020). The costs of achieving a goal include the amount of effort (energy) and time expended. While intrinsic enjoyment, importance, and usefulness contribute positively to the subjective value of a task, perceived cost affects it negatively (Wigfield et al., 2017).

The general goal of general education is to develop mathematical thinking, historical thinking, linguistic thinking, etc. which is the core and meaning of an individual's school preparation for his future professional and personal life in society (Arievitch, 2020). In the context of school education, the teaching of mathematics has a specific position. Mathematical education belongs to the general education of every person, not only because of the high value of the knowledge imparted, but also for its general educational function (Henn, 2001). Students in mathematics classes should not only learn to count but should gain an overview of general relationships. They should know not only how to count, but also why they count like that (Blankertz, 1982). This means that teaching mathematics should primarily pursue goals such as the development of logical thinking and the desire to gain insight into the internal structure of new knowledge. Thus, according to Jordan et al. (2008) students develop self-confidence in their own thinking abilities and a desire for knowledge. Kilpatrick et al. (2001) synthesized research in the field of mathematics education using the concept of mathematical proficiency as the goal of teaching mathematics. Mathematical proficiency has five dimensions:

– Conceptual understanding – understanding of mathematical concepts, operations, and relationships,

– Fluidity of the procedure – skill in performing procedures flexibly, accurately, efficiently, and adequately,

– Strategic competence – the ability to formulate, represent and solve mathematical problems,

– Adaptive thinking – the ability to think logically, reflect, explain, and justify,

– Productive disposition – seeing mathematics as reasonable, useful, and necessary in conjunction with belief in one's own mathematical abilities.

Several research studies have found that teaching mathematics has more or less moved away from its primary goal (e.g. Fuson et al., 2007; Givvin et al., 2009; Gjære, Blank, 2019; Polotskaia, 2022). For example, in an analysis of mathematics lessons in TIMSS 1999 (Givvin et al., 2009), mathematics lessons were described as 'highly algorithmic', 'rule-oriented' and too focused on procedures and rules, with insufficient attention to understanding mathematical concepts with little room for pupils' own thinking. Similarly, according to Wolfram (2010), up to 80 % of mathematics teaching focuses on calculations – algorithms, which are not so crucial in the age of computers. He recommends that the teaching of mathematics should once again focus on the development of skills such as asking the right questions, analysing, creating models, and interpreting results.

Navarro-Ibarra et al. (2017) stated in their work that according to the current didactics of mathematics, mathematics teaching is often based on selling ready-made knowledge and memorizing it, while it should be based on a creative cognitive process with the active participation of learning subjects. Such a focus of mathematics teaching causes students to rely on knowledge of procedures and rules, and these rules and procedures are learned without understanding (Fuson et al., 2007) and not on their own creativity and ability to think correctly. The student is a passive recipient of ready-made recipes for solving individual tasks, this causes a decrease in motivation to learn mathematics as a set of knowledge that needs to be memorized (Escalera-Chávez et al., 2019).

Learning mathematical formulas and rules and memorizing them results in mathematics being complicated for students (Das, 2019) and of little interest because students do not find such skills useful (Pascual, 2022). The deviation of mathematics teaching from the original goal apparently results in a continuous decline in interest in learning mathematics from primary to secondary school (Köller et al., 2001; Frenzel et al., 2012). And it is interest that is an important motivational factor, because it has the function of initiating the desire to achieve a goal that the individual has evaluated as interesting for him. Interest represents a person-goal relationship and is characterized by an individual's involvement in achieving a goal that interested him (Hiddi, Renninger, 2006). Interest theories are based on the assumption that students' individual interest is conditioned by situational interest.

It is assumed that situational interest in mathematics is initiated in interest-dense situations in which students build their own mathematical knowledge in a social environment (Bikner-Ahsbahs, Halverscheid, 2014). For the development of interest in learning mathematics, the fact to what extent the student considers himself capable of learning mathematics plays a key role (Rakoczy et al., 2013; Schukajlow, Krug, 2014). It is this factor that causes a decline in interest in mathematics in higher grades of elementary school and high school. The student gradually finds himself in a social environment where disinterest in mathematics prevails and thus the strength of the situational interest in mathematics also weakens.

We think that the low motivation of students to learn mathematics is due to a lack of intrinsic motivation, because students find it uninteresting. The goal of our research was to find out what kind of mathematics would be interesting for students, i.e. j. how to present mathematics goals to students so that these goals are the primary motivational factor towards the study of mathematics.

3. Methodology

Pedagogical research was carried out in selected secondary schools in Slovakia, always with the consent of the management of the given school, teachers and respondents. The secondary schools whose administrations provided preliminary consent for the research were organized alphabetically by their respective locations. From this list, six secondary schools were selected through simple random sampling using the tool randomnumbergenerator.org.

At the selected schools, all students in the final grades were approached through class teachers. A total of 256 students came to meet with the research team. At this meeting, the students present were familiarized with the content of the research and assured of the overall anonymity of the research. The researchers answered all the questions of the students present, taking care not to influence the reactions of the students during the implementation of the research with their answers. After the initial meeting, 239 students aged 18 and 19 participated in the research itself.

A mixed methods approach was chosen as the research method, which integrates a quantitative questionnaire method with a qualitative interview (Chirumamilla et al., 2020). The aim of the questionnaire research was to find out what type of motivation to learn mathematics prevails among students graduating from high school. According to Fraenkel et al. (2011), it is appropriate to supplement the quantitative questionnaire research with an interview, which allows to gain a deeper insight into the structure of the knowledge obtained from the questionnaire method.

In the quantitative part of the research, a standardized SRQ-Academic questionnaire by Ryan and Connel (1989) was used. This questionnaire is aimed at identifying the motivation and regulation of pupils in the school environment. The questionnaire consists of four parts, each of them is focused on one of the student's activities in the context of school education. Part A is focused on activities related to preparation for teaching. Part B for activities related to the elaboration of tasks that the student receives from the teacher during the lesson. Part C focuses on activities associated with trying to master even more demanding tasks assigned in class. Part D focuses on activities aimed at trying to perform well in class. 8 statements are assigned to each area. These statements saturate the four subdimensions of the student's motivation for the given activity. These motivational (regulatory) dimensions are external motivation, introjected motivation, identified motivation and intrinsic motivation. According to the subjective perception, the student marks the degree of truth of the statement on a four-point Likert scale (1 – disagree, 4 – agree). We used the questionnaire described above in the context of teaching mathematics in order to identify the dominant form of regulation and motivation of the student to learn mathematics.

With the subsequent semi-structured interview, we wanted to identify the difference in the perception of the goal of teaching mathematics between students with predominant intrinsic and extrinsic motivation to learn mathematics. The interview was focused on two basic areas: 1) the expectation of success in mathematics and 2) the subjective value of mathematics, which are the basis of the Erwartungs-Wert-Theorie. Sixteen students participated in the interview, of which eight students with identified predominant external motivation and eight students with identified external motivation. All interview participants agreed to the audio recording. The interview lasted 28 minutes on average. We used a constant comparative method, an inductive coding process (Corbin and Strauss 1990), to obtain data from individual interviews.

To start the experiment, we stated the following research hypothesis:

H: Students' perceived goal of mathematics teaching affects their motivation to learn mathematics.

For the analysis of the research results, we used selected statistical methods, namely methods of descriptive statistics and factor analysis.

4. Data analysis

Before the actual statistical analysis of the data obtained by the questionnaire method, the validity and reliability of the data was first verified. Given that there is a mutual relationship between reliability and validity (good reliability is a necessary condition for proper validity), in our case we calculated reliability to verify the reliability of the data. Cronbach's alpha (e.g. Zumbo et al., 2007) is used to clearly determine the reliability (internal consistency of the test), the calculation of which is part of statistical software. In our case, the value of Cronbach's alpha $\alpha = 0.817$ was calculated using the STATISTICA program. This value points to a strong linear dependence of the questionnaire items (the influence of random errors on the test result is very small), i.e. j. the value $\alpha = 0.817$ confirms the reliability of the obtained data (Cronbach, 1951).

In the next step of the statistical data analysis, we verified whether the respondents who participated in our research can be considered a representative sample with sufficient accuracy. Using the method of principal components, we first determined the number of latent variables – the number of dimensions. Using the STATISTICS program, we obtained a table of eigenvalues of the correlation matrix (Table 1).

Table 1. Eigenvalues of the correlation matrix

Value number	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	8.080	25.250	8.080	25.250
2	4.680	14.626	12.760	39.876
3	3.274	10.230	16.034	50.106
4	2.258	7.057	18.292	57.163
5	1.159	3.622	19.451	60.785
6	1.058	3.307	20.509	64.092

Since, according to Kaiser's criterion, the number of factors should be equal to the number of eigenvalues of the realization of the correlation matrix, which are greater than one, we included only six eigenvalues in Table 1. It shows that this condition is met by six eigenvalues of the correlation matrix, which together (cumulatively) explain 64.092 % of the total variance. In the next step, we performed factor analysis (FA) for 6 and 5 factors. In both cases, we achieved an unsatisfactory solution, because the 5th and 6th factors were saturated by one, respectively, two variables. Based on the above, we decided to use four factors, in accordance with the authors of the SRQ-Academic questionnaire. When performing FA for four factors, we reduced the original thirty-two variables to four latent variables – dimensions. The first dimension explains 25.250 % of the variance contained in the thirty-two observed variables, the second dimension 14.626 %, the third dimension 10.230 % of the variance and the last fourth dimension explains 7.057 % of the variance. The total percentage of explained variance is 57.163 %. The situation is illustrated in Figure 1.

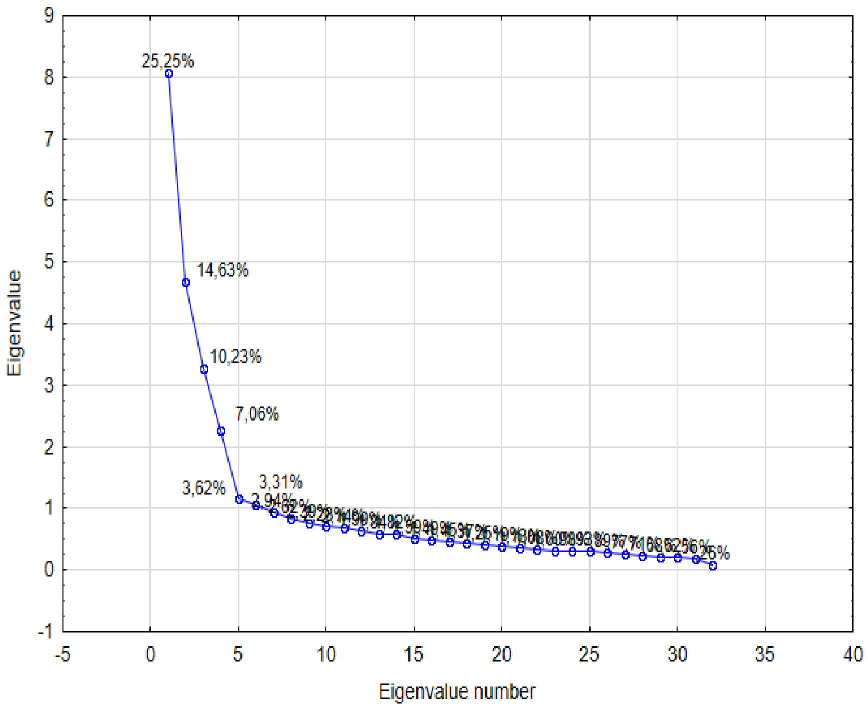

Fig. 1. Correlation matrix eigenvalues and percentage of explained variance

Given that the FA result has a relatively complex structure, for a simpler interpretation of the results it is appropriate to go to the so-called a simple structure in which each factor is highly correlated with (and named after) several variables and its correlations with other variables are low. The transition to a simple structure is made possible by the rotation of the factor scheme – VARIMAX (Osborne, 2015). After the first rotation, we received the following estimate of the matrix of factor loadings (Table 2).

Table 2. Estimation of factor saturation matrix

Factors and items	Factor loading			
	1	2	3	4
Factor 1: Extrinsic motivation				
A. Why do I do my homework?				
2. Because I have a problem if I don't do it.	**0.74**	-0.13	0.13	-0.09
6. Because it's my duty.	**0.78**	-0.05	0.13	-0.15
B. Why do I work on tasks during class?				
9. So the teacher doesn't yell at me.	**0.78**	-0.04	0.12	-0.08
14. Because that's the rule.	**0.76**	-0.03	0.01	-0.23
C. Why do I try to answer difficult questions during class?				
20. Because it's my duty.	**0.80**	-0.04	0.07	-0.15
24. Because I want the teacher to say only good things about me.	**0.75**	-0.04	0.21	-0.13
D. Why do I try to be the best in school?				
25. Because it's my duty.	**0.75**	0.01	0.10	-0.17
28. Because I enjoy working on school tasks as best as I can.	**0.77**	0.03	0.12	-0.09
32. Because I can earn a reward if I do well.	**0.79**	-0.08	0.01	-0.21
Factor 2: Introjected motivation				
A. Why do I do my homework?				
1. Because I want the teacher to think I'm a good student.	0.06	**0.66**	0.02	0.04
4. Because I would feel bad if I didn't do it.	0.09	**0.64**	0.02	0.09
B. Why do I work on tasks during class?				
10. Because I want the teacher to think I'm a good student.	0.03	**0.65**	0.07	-0.02
12. Because I would feel embarrassed if I didn't try.	0.00	**0.67**	0.05	-0.01
C. Why do I try to answer difficult questions during class?				
17. Because I want my classmates to think I'm smart.	0.02	**0.83**	0.03	0.03
18. Because I would feel embarrassed if I didn't try.	0.06	**0.82**	0.02	0.00
D. Why do I try to be the best in school?				
26. Because teachers think I'm a good student.	0.02	**0.69**	0.03	0.07
29. Because I feel bad if I don't do well.	0.10	**0.66**	0.07	0.10
31. I'm proud of myself when I do well in school.	0.06	**0.62**	-0.02	0.15
Factor 3: Identified motivation				
A. Why do I do my homework?				
5. Because I want to understand the subject.	0.16	0.03	**0.76**	0.18
8. Because it's important for me to do my homework.	0.08	-0.02	**0.76**	-0.04
B. Why do I work on tasks during class?				
11. Because I want to learn new things.	0.13	-0.05	**0.74**	0.05
16. Because it's important for me.	0.11	-0.05	**0.79**	0.13
C. Why do I try to answer difficult questions during class?				
21. Because I want to find out if I'm right or wrong.	0.03	-0.03	**0.81**	-0.05
23. Because it's important for me to answer them.	0.10	-0.11	**0.77**	0.11
D. Why do I try to be the best in school?				

Factors and items	Factor loading			
	1	2	3	4
30. Because it's important for me to do well in school.	0.14	0.02	**0.79**	0.24
Factor 4: Intrinsic motivation				
A. Why do I do my homework?				
3. Because it's fun for me.	0.22	0.03	-0.18	**0.74**
7. Because I enjoy doing my homework.	0.20	0.14	-0.14	**0.68**
B. Why do I work on tasks during class?				
13. Because it's fun for me.	0.21	0.05	-0.06	**0.69**
15. Because I enjoy working on tasks during class.	0.19	0.05	-0.11	**0.65**
C. Why do I try to answer difficult questions during class?				
19. Because I enjoy answering difficult questions.	0.24	-0.04	-0.03	**0.72**
22. Because it's fun to answer difficult questions.	0.22	0.09	-0.12	**0.64**
D. Why do I try to be the best in school?				
27. Because I'll have problems if I don't do well in school.	0.20	0.08	-0.15	**0.66**
Eigenvalues	8.08	4.68	3.27	1.16
% of variance	25.3	14.6	10.2	7.06

The use of factor analysis showed that the results obtained by us in Table 2 are consistent with the division of variables into individual factors, which is also declared by the authors of the used questionnaire. Based on the results obtained by statistical analysis, we conclude that the respondents in our research can be considered a representative sample with sufficient accuracy. At the same time, the same division of the questionnaire items into individual factors allows us to keep the original names of the latent variables.

Subsequently, we evaluated the data obtained by questionnaire quantitative research using the Relative Autonomous Index (RAI). First, we calculated the average score in each of the 4 subdimensions for each respondent ($\bar{x}_{Instri}, \bar{x}_{Ident}, \bar{x}_{Intro}, \bar{x}_{Exter}$). Then it applies to RAI

$$RAI = 2 \cdot \bar{x}_{Instri} + 1 \cdot \bar{x}_{Ident} - 1 \cdot \bar{x}_{Intro} - 2 \cdot \bar{x}_{Exter}$$

$RAI < -1{,}5$ value corresponds to prevailing external motivation, $-1{,}5 < RAI < 0$ corresponds to introjected motivation, $0 < RAI < 1{,}5$ corresponds to identified motivation and $RAI > 1{,}5$ corresponds to internal motivation. In Table 3, we present the calculated average score $\bar{x}$ and the average value of the Relative Autonomous Index $\overline{RAI}$ in individual subdimensions. For more details on RAI, see, for example, Grolnick & Ryan (1989).

Table 3. Mean score and mean value of RAI in subdimensions

	External motivation	Introjected motivation	Identified motivation	Intrinsic motivation
$\bar{x}$	2,350	2,511	2,315	1,970
$\overline{RAI}$	-3,584	-0,940	0,861	2,584

The values in Table 3 indicate that external motivation to learn mathematics dominates among the respondents, which was also confirmed by calculating the relative frequency of the dominant motivation among the respondents (Figure 2).

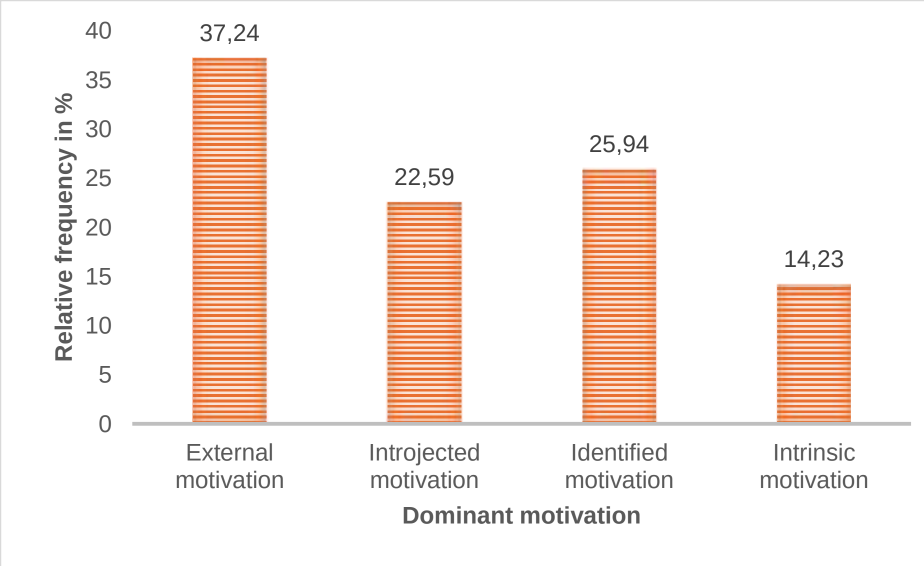

Fig. 2. Relative frequency of dominant motivation

Figure 2 shows that external motivation has the largest relative frequency of dominating motivation (37.27 %). When we add to it the relative frequency of the dominance of introjected motivation (22.59 %), we find that the motivation to satisfy external demands dominates in up to 59.86 % of students learning mathematics. To learn mathematics in order to develop one's own knowledge and skills - internal motivation significantly dominates only 14.23 % of students.

5. Findings from interviews

In the first part of the interview, we found that students consider knowledge and skills in mathematics valuable and necessary. This opinion prevailed not only among students with a dominance of internal motivation, but also among students with external motivation to learn mathematics. Despite this perception of mathematics as a value, most of them expressed that they would not like to study mathematics in their next studies. Almost all students with extrinsic motivation and also three students with predominant intrinsic motivation expressed themselves in this way. Student S6 expressed this fact most succinctly:

"It's great to know such things, but for my future life I already know about enough math."

In particular, students with predominant internal motivation in the interview also indirectly pointed out the reason for trying to avoid mathematics. The students pointed out the discrepancy between the declared goals of teaching mathematics and the goals that, according to them, are pursued by the actual teaching of mathematics. Student S12 expressed himself most clearly in this regard:

"Teachers often tell us why mathematics is necessary in everyday life, but the mathematics we learn seems different to what they say, and frankly, I don't know what it will do for me."

Another key point of intersection in the students' statements was the agreement in the perception of the goal of teaching mathematics. The opinion prevailed among the respondents that the goal of teaching mathematics is to learn to apply memorized calculation algorithms quickly and flawlessly in solving problems. What was interesting in this context was the finding that for students such a goal does not cause demotivation to learn mathematics. According to their statements, they are primarily demotivated by repeating learned procedures in tasks that lack a real context. Student S1 said:

"I calculated a lot of quadratic equations, but what's the point?"

In the second part of the interview, focused on the expected success in mathematics, we found out that students with internal motivation consider solving the assigned task as success, but students with external motivation consider correctly remembering the procedures as success. In this context, it was a remarkable finding that the joy of solving the task is conditioned differently for these groups. Students with external motivation are happy if they managed to choose the correct solution procedure and did not make any numerical error. To illustrate, here is the statement of student S10:

"I'm going to learn the procedure properly and I'm really happy when I manage to use it well."

Intrinsically motivated students experience the joy of solving a task if they managed to solve it on their own with the use of already acquired knowledge.

"I don't enjoy counting routine tasks, but I'm otherwise happy when I figure something out on my own."

From the statements of the students, it follows that for students with external motivation, the joy of solving the assigned task is primarily conditioned by a good result, but for students with internal motivation, the joy of solving the task is mainly conditioned by the quality of the reasoning process, i.e. j. on the way to the result.

During the interview, we also noted the difference between students with predominant internal motivation and students with predominant external motivation in the perception of the time they invest in learning mathematics. Extrinsically motivated students perceive learning mathematics as time-consuming, often much more time-consuming than other subjects. Three of them said that they only learn mathematics as much as they should, because they are not willing to invest more time. Student S3 said.

"It takes me a lot of time to master the procedures, and the result is not always consistent."

Intrinsically motivated students reported spending more time on math when something new is being learned. They spend that time mostly trying to understand new concepts and connecting with already acquired knowledge. After that, they spend less time learning the procedure, because they understand its individual steps. S14 spoke succinctly on this topic.

"When I know what it's about, the procedure often offers itself to me."

6. Discussion

In the quantitative part of the research, we found that 59.83 % of students (respondents) graduating from secondary school had a predominant external motivation to learn mathematics, and the remaining 40.17 % had a predominant internal motivation. However, the use of the basic division of motivation into intrinsic and extrinsic (Middleon, Spanias, 1999) brought a somewhat incomplete insight into the internal structure of the division of students' dominant motivation to learn mathematics. We think that a more faithful picture of the distribution of the dominant motivation to learn mathematics offers the preservation of the original four subdimensions into which the items of the used SRQ-Academic questionnaire were divided. In the case of retaining four subdimensions, external motivation consists of significant external motivation, which was found to be dominant in 37.27 % of students, and introjected motivation, which was dominant in 22.59 % of students. Introjected motivation is characterized as extrinsic motivation with a low level of intrinsic motivation (Ryan, Deci, 2000).

Internal motivation can be divided into intrinsic, i.e. j. intrinsic motivation (Ryan, Deci, 2000). This type of motivation dominated in 14.23 % of students. In addition to the prevailing internal motivation, there is identified motivation, in which the share of internal motivation and external motivation is in favour of the internal one. The identified motivation was dominant in 25.94% of students. Based on these findings, we conclude that the largest part of students learns mathematics based on significant external motivation. Thus, the motive for learning mathematics for them is not to develop their mathematical knowledge and skills, but to satisfy external requirements, such as getting a good evaluation from the teacher or parents. On the other hand, the smallest part of students is motivated to learn mathematics almost exclusively from internal conviction (intrinsic motivation). These students learn mathematics based on their own internal decision in order to improve and develop their mathematical knowledge and skills.

An interesting finding was that there is a relatively large group of students (48.54 %) for whom an exclusive type of dominant motivation was not identified. For these students, one can speak of a "mixed" motivation to learn mathematics. For them, external motivation is "mixed" with internal, while for some external motivation prevails over internal and for others internal motivation prevails over external. We assume that they have a certain internal dynamic between the strength of external and internal motivation. These students may switch from external to internal motivation and vice versa. In the context of other research, one of the factors causing the transition from one form of motivation to another could be the importance of success in mathematics (Herges et al., 2017; Mo, 2019). However, it is not generally possible to say what type of motivation this factor supports. According to Hulleman et al. (2010), striving to get a good grade in mathematics promotes intrinsic motivation for some students and extrinsic motivation for others. Therefore, this factor is considered a neutral motivational factor.

The conducted semi-structured interview brought us additional findings that allowed us to better understand the dynamics of the symbiosis of students' intrinsic and extrinsic motivation to learn mathematics. In the statements of the students, we identified several common features that allowed us to better understand the information obtained by the quantitative part of the research. First of all, we found agreement among students that mathematics is necessary and useful for the development of humanity. On the other hand, lack of interest in mathematics in further studies indicates that mathematics education is not a personal value for most students. According to the students, this discrepancy is caused to a significant extent by the discrepancy between the declared goal of teaching mathematics and the perceived goal of teaching mathematics on the part of the students. This basic finding of ours extends the conclusions of the research carried out by Voica et al. (2020), who suggested that students' level of motivation to learn mathematics is linked to their individual beliefs.

It follows from the students' statements that the goal of teaching mathematics is the quick and flawless application of memorized procedures. And the fulfilment of this goal has a demotivating character for students, in several aspects. Students with a dominant external and also students with a dominant internal motivation to learn mathematics agreed that a strong demotivator for them is the "endless" practice of calculation procedures, even on tasks "detached" from reality, which corresponds to the finding (Boaler, 2015). Although both groups agreed in identifying a strong demotivator, the causes of demotivation are different. In the first group, the cause is the repetition of a procedure they already master. In the second group, the reason is the large amount of time they need to gradually remember the procedure. Several students (mainly from external motivation) indicated that a good performance in mathematics is not decisive for them personally, therefore they focused on learning the minimum necessary to achieve a "good" grade for them. At the same time, their motivation to learn mathematics decreases with the increasing time required to learn increasingly complex procedures with uncertainty of success. In further research, it would be necessary to investigate whether reducing the feeling of failure in mathematics by correctly approaching students' mistakes will slow down the decline in motivation to learn mathematics.

In contrast, students with a strong dominance of intrinsic motivation see good performance in mathematics as a sign that they "think" well and are therefore motivated to learn mathematics. The decrease in their motivation to learn mathematics is smaller than that of students with external motivation. It emerged from their statements that this is primarily caused by the decreasing correlation between teaching mathematics and their personal interest.

Based on our research, we conclude that a key role in the dynamics of the symbiosis of external and internal motivation is played by the factor: The importance of good performance for oneself. This factor is closely connected with the perceived goals of teaching mathematics. If the student perceives the set goal as important for him, it is motivating for him to achieve a good performance in achieving it. It follows from the statement of the students that this connection could be one of the foundations of the emergence of the dominance of internal motivation to learn mathematics over external. Their internal motivation gains even more advantage if they manage to achieve the desired performance. Internally motivated students considered the ability to solve problems independently, based on already acquired knowledge, to be a good performance. This finding of ours complements the knowledge that students associate understanding with greater liking of mathematics (Wilkie, Sullivan, 2018).

7. Conclusion

From the first – quantitative part of the research, it emerged that students are mostly dominated by an exclusively external motivation to learn mathematics. The form of dominant motivation is strongly influenced by the goal of teaching mathematics, which is perceived by the students. If the goal of teaching mathematics corresponds to their personal value, then they are motivated to achieve this goal. A goal perceived as personally useful, coupled with the personal value of performing well in achieving it, supports intrinsic motivation to learn mathematics. Students with dominant internal motivation do not experience the increasing time-consuming nature of learning mathematics as negatively as students with external motivation.

8. Acknowledgments

This work was also supported by the Scientific Grant Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic and the Slovak Academy of Sciences under contract No. KEGA 019 ŽU-4/2023.

References

Arievitch, 2020 – *Arievitch, I.M.* (2020). Reprint of: The vision of Developmental Teaching and Learning and Bloom's Taxonomy of educational objectives. *Learning, culture and social interaction*. 27: 100473.

Bakker, Akkerman, 2014 – *Bakker, A., Akkerman, S.F.* (2014). A boundary-crossing approach to support students' integration of statistical and work-related knowledge. *Educ Stud Math*. 8: 223-237.

Bikner-Ahsbahs, Halverscheid, 2014 – *Bikner-Ahsbahs, A., Halverscheid, S.* (2014). Introduction to the theory of interest-dense situations (IDS). *Networking of theories as a research practice in mathematics education*. Pp. 97-113.

Blankertz, 1982 – *Blankertz, H.* (1982). Die Geschichte der Pädagogik: von der Aufklärung bis zur Gegenwart. Verlag Büchse d. Pandora.

Boaler, 2015 – *Boaler, J.* (2015). *Mathematical mindsets*. San Francisco: Jossey-Bass, A Wiley Brand.

Corbin, Strauss, 1990 – *Corbin, J.M., Strauss, A.* (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative sociology*. 13(1): 3-21.

Das, 2019 – *Das, K.* (2019). Role of ICT for better mathematics teaching. *Shanlax International Journal of Education*. 7(4): 19-28.

Di Martino, Zan, 2011 – *Di Martino, P., & Zan, R.* (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *Zdm*. 43(4): 471-482.

Eccles, Wigfield, 2020 – *Eccles, J. S., Wigfield, A.* (2020). From expectancy-value theory to situated expectancy-value theory: a developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology*. 61, Article 101859.

Eccles, 2005 – *Eccles, J. S.* (2005). Subjective task value and the Eccles et al. model of achievement-related choices. In A. J. Elliot & C. S. Dweck (Hrsg.). Handbook of competence and motivation (Pp. 105-121). New York: Guilford.

Eccles et al., 1983 – *Eccles, J.S., Adler, T.F., Futterman, R., Goff, S.B., Kaczala, C.M., Meece, J.L., Midgley, C.* (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Hrsg.). Achievement and achievement motives (Pp. 74-146). New York: W. H. Freeman.

El-Adl, Alkharusi, 2020 – *El-Adl, A., Alkharusi, H.* (2020). Relationships between self-regulated learning strategies, learning motivation and mathematics achievement. *Cypriot Journal of Educational Sciences*. 15(1): 104-111.

Elliot, 1999 – *Elliot, A.J.* (1999). Approach and avoidance motivation and achievement goals. *Educational psychologist*. 34(3): 169-189.

Elliot, Murayama, 2008 – *Elliot, A.J., Murayama, K.* (2008). On the measurement of achievement goals: critique, illustration, and application. *Journal of educational psychology*. 100(3): 613.

Escalera Chávez et al., 2019 – *Escalera Chávez, M.E., Moreno García, E., Rojas Kramer, C.A.* (2019). Confirmatory Model to Measure Attitude towards Mathematics in Higher Education Students: Study Case in SLP Mexico. *International Electronic Journal of Mathematics Education*. 14(1): 163-168.

Foster, N., & Schleicher, 2022 – *Foster, N., Schleicher, A.* (2022). Assessing creative skills. *Creative Education*. 13(1): 1-29.

Fraenkel et al., 2011 – *Fraenkel, J.R., Wallen, N.E., Hyun, H. H.* (2011). How to design and evaluate research in education: New York: McGraw-Hill Humanities/Social Sciences/Languages, ISBN: 978-0-07-809785-0.

Frenzel et al., 2012 – *Frenzel, A.C., Pekrun, R., Dicke, A.L., Goetz, T.* (2012). Beyond quantitative decline: conceptual shifts in adolescents' development of interest in mathematics. *Developmental psychology*. 48(4): 1069.

Fuson et al., 2005 – *Fuson, K. C., Kalchman, M., & Bransford, J. D.* (2005). Mathematical understanding: An introduction. *How students learn: History, mathematics, and science in the classroom*. Pp. 217-256.

Gainsburg, 2015 – *Gainsburg, J.* (2015). Engineering students' epistemological views on mathematical methods in engineering. *Journal of Engineering Education*. 104(2): 139-166.

Givvin et al., 2009 – *Givvin, K. B., Jacobs, J., Hollingsworth, H., Hiebert, J.* (2009). What is effective mathematics teaching? International educators' judgments of mathematics lessons from the TIMSS 1999 Video Study. In J. Cai, G. Kaiser, B. Perry, N.-Y. Wong (Eds.), Effective mathematics teaching from teachers' perspectives: National and cross-national studies (pp. 37-69). Boston, MA: Sense.

Gjære, Blank, 2019 – *Gjære, Å.L., Blank, N.* (2019). Teaching mathematics developmentally. *For the Learning of Mathematics*. 39(3): 28-33.

Grolnick, 1989 – *Grolnick, W.S., Ryan, R.M.* (1989). Parent styles associated with children's self-regulation and competence in school. *Journal of educational psychology*. 81(2): 143.

Henn, Kaiser, 2001 – *Henn, HW., Kaiser, G.* (2001). Mathematik – ein polarisierendes Schulfach. *ZfE*. 4: 359-380.

Herges et al., 2017 – *Herges, R.M., Duffied, S., Martin, W., Wageman, J.* (2017). Motivation and achievement of middle school mathematics students. *The Mathematics Educator*. 26(1).

Hidi, Renninger, 2006 – *Hidi, S., Renninger, K.A.* (2006). The four-phase model of interest development. *Educational psychologist*. 41(2): 111-127.

Hoyles et al., 2010 – *Hoyles, C., Noss, R., Kent, P., Bakker, A.* (2010). Improving mathematics at work: The need for techno-mathematical literacies. Routledge.

Hulleman et al., 2010 – *Hulleman, C.S., Schrager, S.M., Bodmann, S.M., Harackiewicz, J.M.* (2010). A meta-analytic review of achievement goal measures: Different labels for the same constructs or different constructs with similar labels? *Psychological bulletin*. 136(3): 422.

Chirumamilla et al., 2020 – *Chirumamilla, A., Sindre, G., Nguyen-Duc, A.* (2020). Cheating in e-exams and paper exams: the perceptions of engineering students and teachers in Norway. *Assessment & Evaluation in Higher Education*. 45(7): 940-957.

Jordan et al., 2008 – *Jordan, A., Krauss, S., Löwen, K., Blum, W., Neubrand, M., Brunner, M., ... , Baumert, J.* (2008). Aufgaben im COACTIV-Projekt: Zeugnisse des kognitiven Aktivierungspotentials im deutschen Mathematikunterricht. *Journal für Mathematik-Didaktik*. 29: 83-107.

Kilpatrick et al., 2001 – *Kilpatrick, J., Swafford, J., Findell, B.* (2001). *Adding it up: Helping children learn mathematics* (Vol. 2101). National research council (Ed.). Washington, DC: National Academy Press.

Köller et al., 2001 – *Köller, O., Baumert, J., Schnabel, K.* (2001). Does interest matter? The relationship between academic interest and achievement in mathematics. *Journal for research in mathematics education*. 32(5): 448-470.

Locke, Latham, 2013 – *Locke, E.A., Latham, G.P.* (Eds.). (2013). *New developments in goal setting and task performance* (Vol. 24, p. 664). New York: Routledge.

Locke, Latham, 2002 – *Locke, E.A., Latham, G.P.* (2002). Building a practically useful theory of goal setting and task motivation: A 35-year odyssey. *American Psychologist*. 57(9): 705-717.

Maass et al., 2019 – *Maass, K., Geiger, V., Ariza, M.R., Goos, M.* (2019). The role of mathematics in interdisciplinary STEM education. *Zdm*. 51: 869-884.

Marsh et al., 2019 – *Marsh, H.W., Pekrun, R., Parker, P.D., Murayama, K., Guo, J., Dicke, T., Arens, A.K.* (2019). The murky distinction between self-concept and self-efficacy: beware of lurking jingle-jangle fallacies. *Journal of Educational Psychology*. 111: 331-353.

Middleton, Spanias, 1999 – *Middleton, J.A., Spanias, P.A.* (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for research in Mathematics Education*. 30(1): 65-88.

Mo, 2019 – *Mo, J.* (2019). How Is Students' Motivation Related to Their Performance and Anxiety? PISA in Focus. No. 92. *OECD Publishing*.

Navarro-Ibarra et al., 2017 – *Navarro-Ibarra, L., García-Santillán, A., Cuevas-Salazar, O., Ansaldo-Leyva, J.* (2017). Mathematics, Technology and Learning: How to Align These Variables in Order to Explain Anxiety Towards Mathematics and Attitude Towards the Use of Technology for Learning Mathematics. *EURASIA Journal of Mathematics, Science and Technology Education*. 13(9): 6211-6229.

Niss, 2015 – *Niss, M.* (2015). Mathematical competencies and PISA. In *Assessing mathematical literacy* (pp. 35-55). Springer, Cham.

Osborne, 2015 – *Osborne, J.W.* (2015). What is rotating in exploratory factor analysis? *Practical Assessment, Research, and Evaluation*. 20(1).

Pascual, 2022 – *Pascual, E.A.* (2022). Getting the answer exactly right: dealing with math misconception. *Getting the Answer Exactly Right: Dealing with Math Misconception*. 93(1): 5-5.

Pepin et al., 2021 – *Pepin, B., Biehler, R., Gueudet, G.* (2021). Mathematics in engineering education: A review of the recent literature with a view towards innovative practices. *International Journal of Research in Undergraduate Mathematics Education*. 7(2): 163-188.

Polotskaia et al., 2022 – *Polotskaia, E., Fellus, O. O., Cavalcante, A., Savard, A.* (2022). Students' problem solving and transitioning from numerical to relational thinking. *Canadian Journal of Science, Mathematics and Technology Education*. 22(2): 341-364.

Rakoczy et al., 2013 – *Rakoczy, K., Harks, B., Klieme, E., Blum, W., Hochweber, J.* (2013). Written feedback in mathematics: Mediated by students' perception, moderated by goal orientation. *Learning and Instruction*. 27: 63-73.

Rose, Betts, 2004 – *Rose, H., Betts, J.R.* (2004). The effect of high school courses on earnings. *Review of Economics and Statistics*. 86(2): 497-513.

Rønning, 2004 – *Rønning, F.* (2022). Learning Mathematics in a Context of Electrical Engineering. In: Biehler, R., Liebendörfer, M., Gueudet, G., Rasmussen, C., Winsløw, C. (eds) Practice-Oriented Research in Tertiary Mathematics Education. Advances in Mathematics Education. Springer, Cham.

Ryan, Connell, 1989 – *Ryan, R.M., Connell, J.P.* (1989). Perceived locus of causality and internalization: Examining reason for acting in to domains. *Journal of Personality and Sociál Psychology*. 57: 749-761.

Ryan, Deci, 2000 – *Ryan, R.M., Deci, E.L.* (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology*. 25: 54-67.

Schukajlow, 2015 – *Schukajlow, S.* (2015, February). Is boredom important for students' performance?. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1273-1279).

van der Wal et al., 2017 – *van der Wal, N. J., Bakker, A., Drijvers, P.* (2017). Which techno-mathematical literacies are essential for future engineers? *International Journal of Science and Mathematics Education*. 15: 87-104.

Voica et al., 2020 – *Voica, C., Singer, F. M., Stan, E.* (2020). How are motivation and self-efficacy interacting in problem-solving and problem-posing? *Educational Studies in Mathematics*. 105(3): 487-517.

Weber, 2003 – *Weber, K.* (2003). The relationship of interest to internal and external motivation. *Communication Research Reports*. 20(4): 376-383.

Wigfield, 2017 – *Wigfield, A., Rosenzweig, E.Q., Eccles, J.S.* (2017). Achievement values: interactions, interventions, and future directions. In A. J. Elliot, C. S. Dweck & D. S. Yeager (Hrsg.), *Handbook of competence and motivation: theory and application* (Pp. 116-134). New York: Guilford.

Wilkie, Sullivan, 2018 – *Wilkie, K.J., Sullivan, P.* (2018). Exploring intrinsic and extrinsic motivational aspects of middle school students' aspirations for their mathematics learning. *Educational Studies in Mathematics*. 97: 235-254.

Wolfram, 2010 – *Wolfram, C.* (2010). Teaching kids real math with computers. [Electronic resource]. URL: http://www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_computers.html

Wood, Locke, 1990 – *Wood, R., Locke, E.* (1990). Goal setting and strategy effects on complex tasks. In B. Staw & L. Cummings (Eds.), Research in organizational behavior (Vol. 12, pp. 73-109). Greenwich, CT: JAI Press.