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The past decade has witnessed a period of explosive development in information technologies (IT), and the related issue of the changing nature of the teacher's role within modern educational systems has become central. Today's teachers are unable to ignore the educational potential of state-of-the-art information technologies, or the hardware and software advances, which are shifting the process of education towards the next level on all fronts. Based on existing and ongoing research into teaching and learning with technology, teachers are able to considerably increase the potential for higher educational impact of lessons, and for increased student engagement and motivation.

Following the introduction of computer science courses, there were several attempts to bring computers into the tutoring or self-study process in various disciplines. To achieve this goal, in the early stages, students were asked to use simple, in-house computer training programs, and the entire process was managed by the IT teacher. Beyond this, enthusiastic subject-based teachers would sometimes take part, using certain software programs with which they were familiar to reinforce the learning in their respective courses. But generally speaking, most attempts to fully implement computers or technology across the curriculum (i.e., system-wide, or departmental implementation) have failed. The obstacles for wider implementation have included the following: the imperfection of the software products; the organizational difficulties related to computer lab access; and the unpreparedness of teachers in facilitating technology-based lessons.

Psychologically, the process of implementing information technologies into education was made significantly less daunting by the increasing user-friendliness and overall appearance of the courseware packages and programs. Further, information technologies made it increasingly easier to access information (e.g., World Wide Web) to be used in learning opportunities, to personalize the learning, and to differentiate instruction according to student needs and strengths. Furthermore, information technologies allowed for new kinds of student interaction (internal and external to the educational institution), and new ways of encouraging active student involvement. However, due to the organizational and methodical difficulties described above, the ambitious goals regarding technology in education have by in large still not been fully realized in schools.

Today, we do however observe a significant growth in teachers' interest in the application of information technologies in education, especially in the domain of mathematics. Although there have been a number of dynamic geometry programs widely used in education throughout the past few decades (e.g., Autograph, Cabri, The Geometer's Sketchpad), one relatively new software package, GeoGebra, has seen a tremendous increase in international popularity and usage.

GeoGebra (geogebra.org) is a freely-available, open-source, dynamic mathematics software package which began as a masters' thesis project created by Austrian professor Markus Hohenwarter in 2002. After 10 years, some 35 developers and a small army of volunteer contributors and translators have made it one of the most useful and popular educational programs in the world. In 2012, GeoGebraTube was released with version 4.0 of the software as a simplified platform for applet sharing among international users. In 2012, GeoGebraWeb, an Html 5 version of the program which runs in a web browser, was released to the public. This version of the software can be used on desktop/laptop computers, tablets, or even smartphones.

Currently, developers are working on a tablet version of GeoGebra for iPad, Android, and Microsoft Windows 8. GeoGebra has been translated into more than 44 languages, used in 190 countries, and downloaded by approximately 500,000 users per month, and so it is clearly making an impact on mathematics education. The real strength of the software development lies in the strength and energy of its extended and enthusiastic international community (Hohenwarter, Jarvis, & Lavicza, 2009).

In this special issue of the *European Journal of Contemporary Education*, "**GeoGebra in the Digital Era**," we will be presenting five articles relating to the implementation of, and research surrounding, the mathematical software known as GeoGebra.

In the first article, *Teaching Materials "Surface Area of Geometric Figures," Created Using the Software Package GeoGebra,* author Slaviša Radović maintains that in order to increase student engagement and motivation, teachers must invite the use of multimedia and change their teaching and assessment practices accordingly. Radović provides the reader with an example of the power of GeoGebra in allowing students to model and explore concepts relating to **surface area**, with a particular focus on increasing interactivity between students and teachers in the learning process.

In the second article, *The First Derivative of an Exponential Function with the "White Box/Black Box" Didactical Principle and Observations with GeoGebra,* authors Natalija Budinski and Stephanie Subramaniam demonstrate how GeoGebra can be used to experiment, visualize, and connect various concepts such as **function, first derivative, slope, and tangent line**. According to Budinski and Subramaniam, GeoGebra enriches the educational process and opens up new questions and possibilities for students, as instructors embrace new didactical approaches.

In the third article, *Modeling and Visualization Process of the Curve of Pen Point by GeoGebra*, authors Muharrem Aktümen, Tuğba Horzum, and Tuba Ceylan provide a specific example of how GeoGebra can be used to model a mathematics topic, in this case the use of **parametric equations**, using both two- and three-dimensional visualization. The authors contend that determining mathematical concepts and relationships based on real-life models using these types of technology-based tasks, as well as jointly considering the algebraic and geometric representations during the process, improves the student learning and deepens mathematical understanding.

In the article, *GeoGebra Software Use within a Content and Language Integrated Learning Environment,* Helena Binterová and Marek Šulista present the results of a research study focusing on the analysis, comparison, and description of students' attitudes towards the **teaching of mathematics lessons presented in a foreign language (English)** using the Content and Language Integrated Learning (CLIL) method in three elementary schools. The authors highlight the difference between the attitudes of the CLIL method learners and those of their student counterparts who experienced similar mathematics lessons, but in their mother tongue (Czech). The research also focused on the question of whether or not, or to what degree, the implementation of the foreign language (English) along with the use of an interactive tool, such as GeoGebra software in mathematics lessons, was perceived as being meaningful and as significantly improving the effectiveness of student learning.

In the final article entitled, *Creating a YouTube-Like Collaborative Environment in Mathematics: Integrating Animated GeoGebra Constructions and Student-Generated Screencast Videos,* researchers Jill Lazarus and Geoffrey Roulet discuss the integration of student-generated GeoGebra applets and Jing screencast videos to create a YouTube-like medium for **online sharing** in mathematics. They underscore the value of combining dynamic mathematics software and screencast videos for facilitating communication and representations in a digital context, and they specifically highlight the power of GeoGebra software for student expression and creativity.

As we have seen, the first three articles describe the effectiveness of GeoGebra, and some related technology-rich pedagogical approaches, at the elementary, secondary, and tertiary levels, respectively. The final two articles report on research studies in which GeoGebra was used to enhance mathematics learning in the English language within Czech schools using the CLIL method; and, to encourage creativity and communication among students as they shared GeoGebra constructions and screencast videos within an online social media forum.

We hope you will agree that this special issue provides an interesting and informative series of perspectives on both teaching and research in relation to the software known as GeoGebra, as it continues to be adopted and further developed in the 21st century digital era.

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Teaching Materials "Surface Area of Geometric Figures," Created Using the Software Package *GeoGebra*

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Abstract. Social development, the progress of technology, and changing economic forces certainly affect the development of the current educational system. One of the main problems of today's school system is how to maintain focus, concentration, and interest among students with regards to the learning that takes place during classes. An important feature of modern teaching is multimedia. However, the use of multimedia requires a certain transformation in the teaching process. Given the fact that the focus of the teaching process has been shifting away from the curriculum and the teacher, and towards the student, multimedia will undoubtedly contribute in a significant way to the modernization of traditional teaching. It is not unrealistic to think that technology will become a regular part of the daily routines of teaching. In this work, an innovative approach to teaching mathematics in elementary and high schools through the use of the software known as GeoGebra is introduced. This approach is demonstrated using the example of 'surface area' as a mathematical concept, wherein the goal was to increase interactivity between teachers and students, and to improve the quality of teaching.
Keywords: teaching mathematics; GeoGebra; modern education; interactivity

1. Introduction

The future of the school cannot be understood without understanding its development and history (Cross, 2004). One of the key needs of the human race is education, which is reflected in the maintenance of the species. In order to fulfil that need, adult community members have cared about youth. As the knowledge and skills that were to be transferred to the next generation kept accumulating, it was clear that the care of young people had to stand apart from everyday life, and needed to be organized differently. Over time, education and teaching the young have become a special social activity, one that has gradually been formalized and institutionalized. One characteristic of the formal organization of youth education was the genuine concern about society's survival and the maintenance of the human species.

The school is one of the oldest institutions of known societies, dating back approximately 7000 years to Sumeria and ancient China. Education continued to formally develop through the Greek and Roman cultures, was influenced greatly by various world religions and international conflicts, and has continued to evolve during the modern era. The school is able to survive so long because it is constantly changing, adapting, and harmonizing its work with the needs and demands of the communities within which it has existed. The school has changed and continued to take on new forms, always transforming into a new school, one that meets the needs of the community.

For all those concerned with education in any way, it is of great interest to understand how schooling will play a social role in the future of modern society. Social development, the progress of technology, and changing economic forces certainly affect the development of the current educational system. The problems that we will encounter in the future, the direction of technical advances, the development of economic systems can no longer be predicted using past experience. Currently, we cannot even be sure of what the world will look like in ten or twenty years—the world in which these students that we are teaching today, will work, live, and raise families.

Our goal as teachers is to prepare students properly to face problems that await them in the future. Unlike the previous school system, where it was required to be submissive and obedient to reproduce the stored or known information, we now ask students to understand, to think, and to choose the correct answers. We must value creativity and freedom of thought, which has often been neglected in formal schooling.

Before we start talking about the development of the current education system, we must look at its shortcomings. Perhaps the greatest example is the lack of attention and concentration demonstrated by students during the average lesson. The goal of every teacher is to make lessons fun and interesting enough so that more students will be motivated to be attentive during classes, to think, and to follow the material that is being presented (Ruthven, Hennessy, & Brindley, 2004). However, when it comes to mathematics classes, especially with abstract mathematical topics, this is not an easy task. Teachers can use the most modern technology and educational software to present abstract mathematical concepts (Polya, 1962) in a virtual environment (e.g., using computer software) in which students feel very comfortable. In this way, the space where students are accustomed to playing and having fun is turned into a space where in students can be free to explore and to learn (Pećanac, Lambić, & Marić, 2011).

Classes were formerly designed to fit the needs of the *average student*, and this was another drawback of the traditional education system. The use and implementation of multimedia and educational software classes may significantly contribute to the modernization and individualization of learning (Prawat, 1992). Educational software that is designed with good didactic and methodical preparation of materials is able to better correspond with the existing knowledge, abilities, and skills of each student. Instruction is adjusted to each student in light of their individual needs, since the progress of students does not have to be tied to the overall progress of the class, but rather to their own work and learning opportunities. The focus of the teaching process has indeed shifted away from teachers and teaching materials, towards the student and learning materials. One might say that the student is finally in the centre of our teaching.

GeoGebra is a mathematical software developed by Markus Hohenwarter for teaching mathematics in schools. It was originally created as part of his graduate studies (Masters), and has since become a very popular software package that is used around the world. Hohenwarter continued to improve the program through the completion of his doctoral dissertation. He has won a number of European and international awards in the field of educational software. GeoGebra is a software package that connects algebra, geometry, and analysis—this combination forms the basis of its name (i.e., geometry + algebra). It is a freely-available and open-source software, and it is currently available in over 50 languages. User guides, a user forum, and many useful related resources are also available through the main website (http://www.geogebra.org/). GeoGebra is written in Java, and as such can be run in any web browser. GeoGebra combines two different approaches of visualizing mathematical objects. More precisely, geometry and algebra are equally and dynamically represented, i.e., it is possible to assign objects to equations, and then to change the graphic objects and observe how the equations change, or vice versa.

One can use GeoGebra to create interactive web pages, so-called interactive worksheets, and the easy publication of these drawings to the web is yet another benefit of the software. Without knowledge of HTML programming or how to create web pages, one can easily use the GeoGebra "Wizard" that requires only a few clicks and the entering of basic information related to the activity. This method allows students to better interact with the material, as well as providing a robust and reliable method of sharing files with their peers in the classroom, school, and beyond. As a result, students are often more engaged and interested in their mathematical work (Hohenwarter & Preiner, 2007). Interactivity of applets in web pages is increased if Javascript buttons are present on the site, allowing the interaction of text and applets. By preparing these kinds of interactive worksheets, teachers can capitalize on a powerful tool that can bring abstract mathematical concepts to life for students of all ages (Chen-Wo, Jiann-Min, Quo-Ping, & Maiga, 2009).

2. Interactive Educational Activity

In this section, an interactive educational activity for the elementary and high school level is presented, related to the term "surface area" of a figure.[*] Dynamic interactivity is achieved by using JavaScript and PHP functions, and a

[*] The activity is available for downloading in the form of GeoGebra applets at the website: http://alas.matf.bg.ac.rs/~ml06125

MatJax function enables the writing of mathematical text and formulas using standard LaTex commands. All educational materials are written primarily for students (students can learn independently, expanding their knowledge and making gains in individual progress) but also for teachers (including ideas for multi-media classes). All of the available activities are divided into categories based on various mathematical topics relating to the surface area of figure, and according to the educational standards issued by the Ministry of Education of the Republic Serbia. Each category contains four different sections of interactive materials that are mutually complementary.

The first section related to the aspect of learning. By using interactive GeoGebra applets (see Figures 1-3) and dynamic web pages, geometric figures/objects and their basic properties are represented to students in interesting ways, as they learn to calculate the surface areas of a figure.

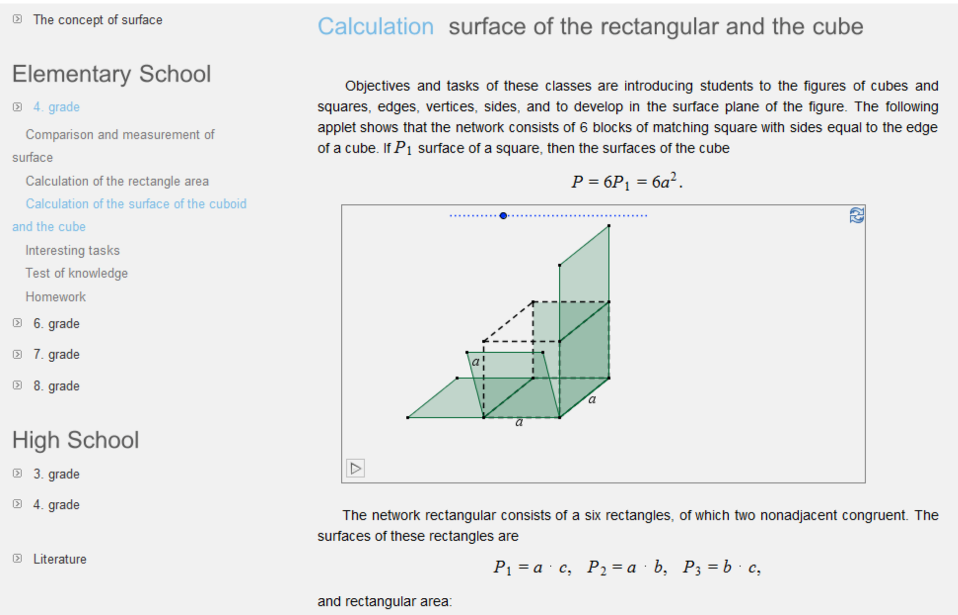

Figure 1. Grade 4 activity: Surfaces of the cube

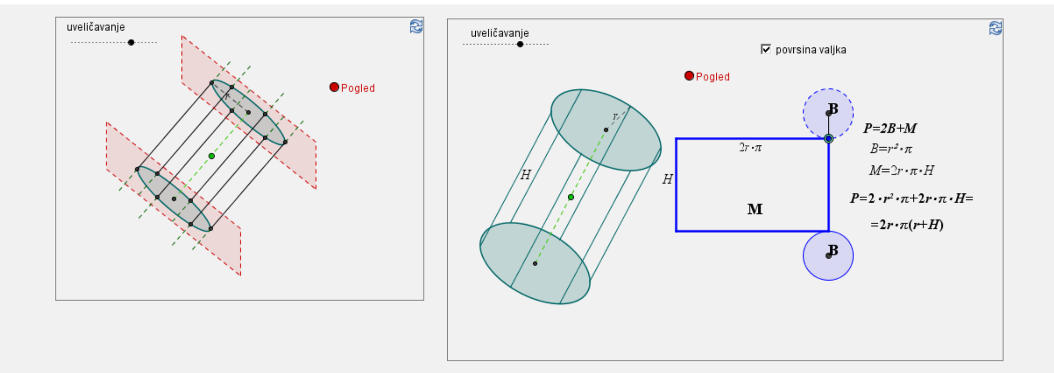

Figure 2. Grade 8 activity: Area of the cylinder

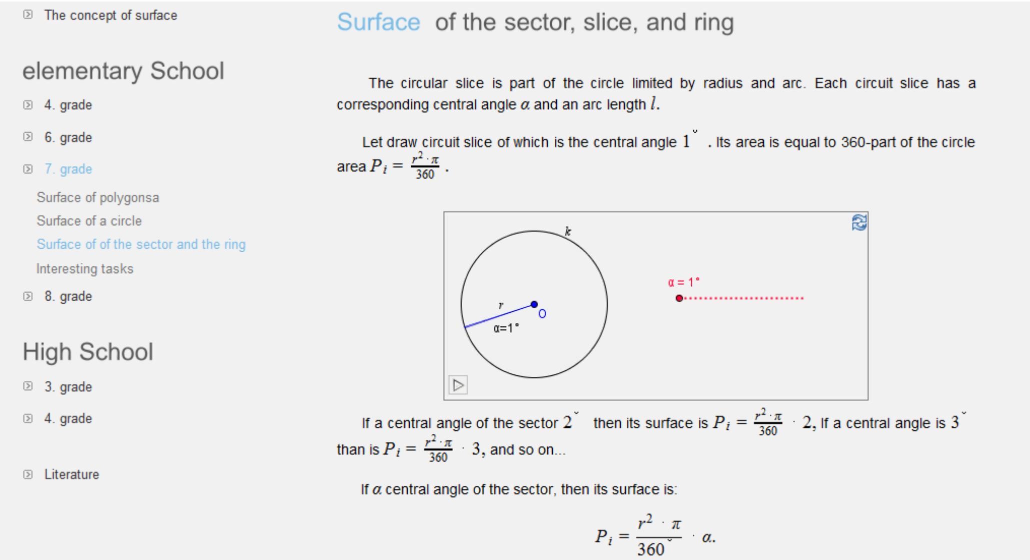

Figure 3. Grade 7 activity: Surface area of the sector, slice, and ring

The main feature of this presentation of educational material is that it allows students to individually explore and discover relationships between observed objects.

By moving objects, points, and the slider on the applets, students notice changes that occur, and in this way draw their own conclusions. In every Web page with applets included, there is mathematical text that introduces students to the event that takes place on the applet, and students can later check to see if they have discovered the main conclusion of that particular lesson.

The second part refers to the tasks for exercises. When students learn the material that concerns the properties of geometric objects and the ways in which one computes the surface area of figures, the next step is to explore the learning further with problem assignments. By opening the link "Interesting tasks" (Figure 4) within each class, we get a web page with the problems that need to be solved. The problems are divided into several groups, depending on the level of difficulty or the type of task. If students have some problems while solving the tasks, they may simply click on the "solution" button that appears below each task. This button opens a field in which the steps of the solution are explained in detail and the exact results are given. This type of initiation of the problem allows students to solve tasks independently, and (if needed) simultaneously check if are they working well or if there is another, perhaps simpler, way of solving the problem.

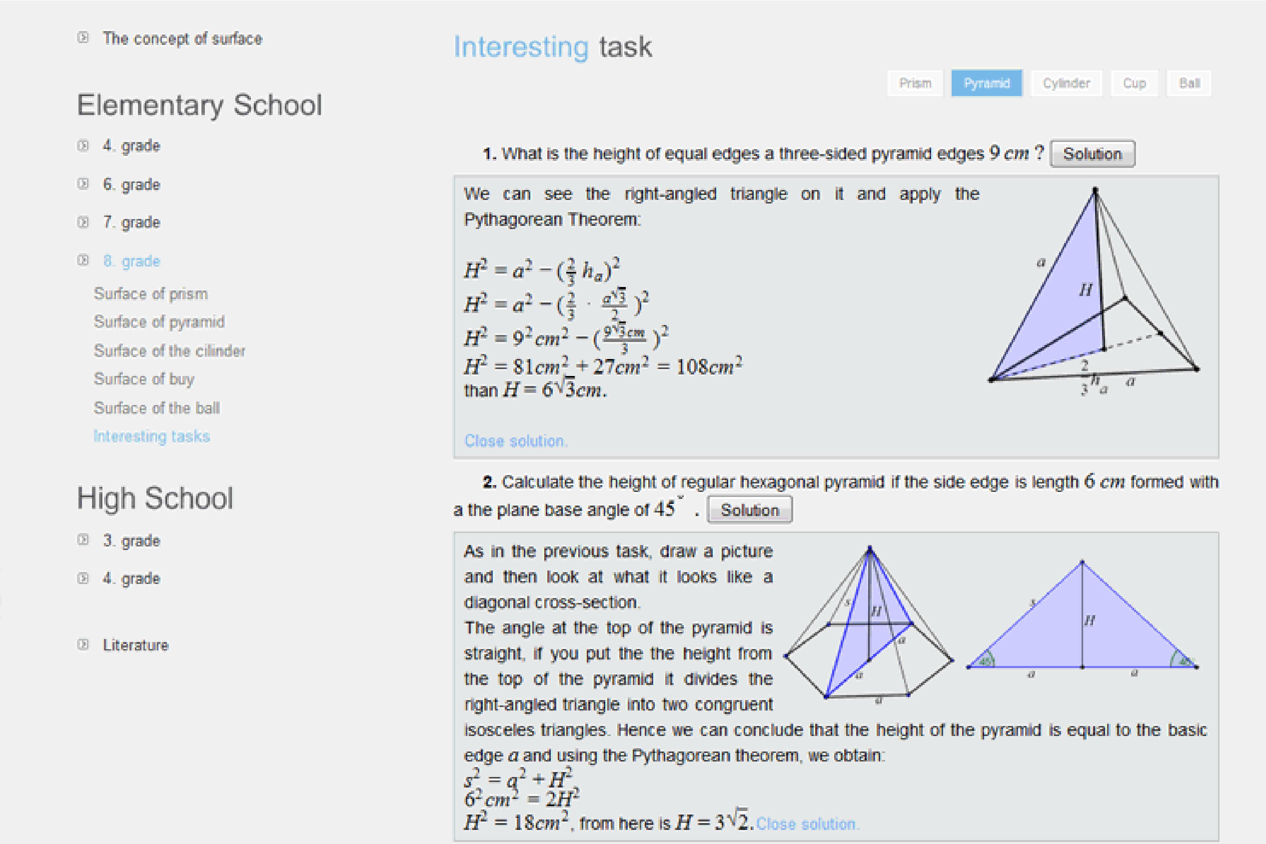

Figure 4. Interesting tasks

The third section is related to the self-test regarding knowledge (Figure 5). Students who have successfully completed tasks in the part of the site dedicated to the activity can further test their knowledge by solving a related test. By clicking on the "Test" button within each class, one can open the web page that features the instructions for students and the selection of the level assignment difficulty. Tasks are grouped in three tests, arranged by difficulty, as instructed by "educational standards for the end of compulsory education for mathematics", according to the Ministry of Education and the Institute for the Evaluation of Education of the Republic of Serbia. When the test is opened, the measuring of time begins.

At the end of each question, there is a place to enter the correct answer, and the number of points awarded for each part of the question is also indicated. When a student is done with the task, by clicking on the "Check" button, his answers are evaluated and he accumulates a certain number of points. The student can win a total of up to 100 points. The "Check" button opens a new page where all of the tasks are reviewed and where the student can read information about the test, such as which tasks are correct, the number of points that were won, the time spent solving the test, and the exact solutions for the tasks (in order to realize their mistakes).

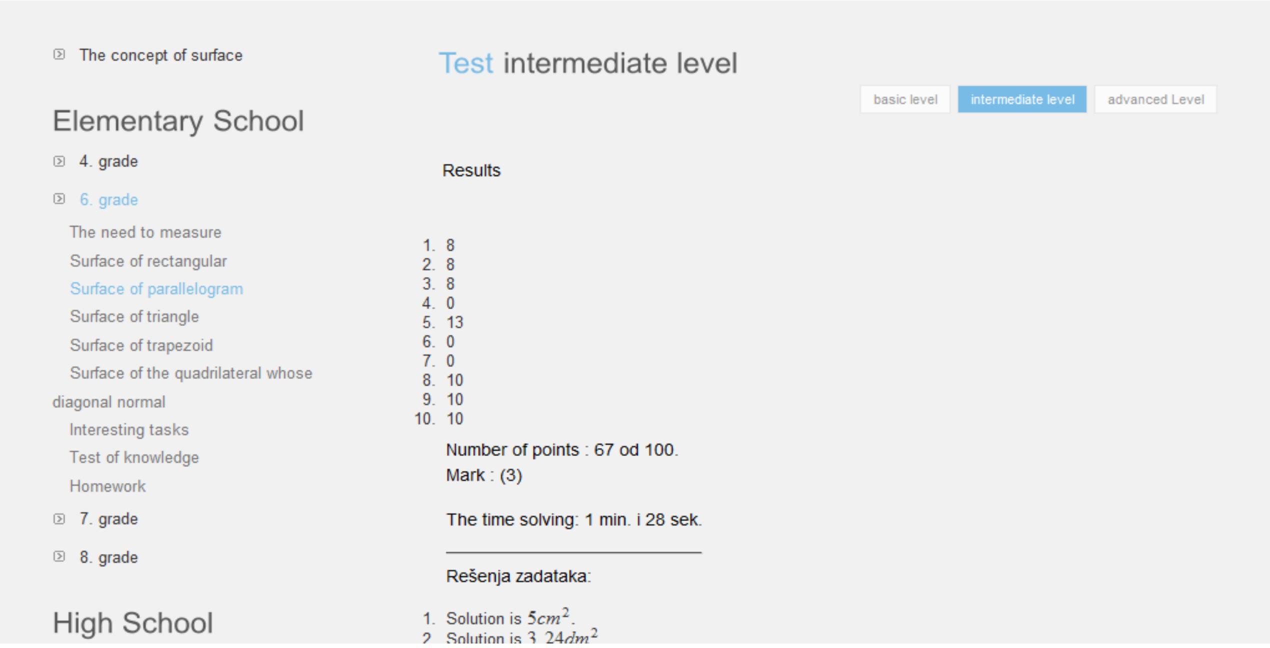

Figure 5. Test of knowledge

The fourth part refers to homework (Figure 6). This part of the site is dedicated to students who work at home and whose knowledge teachers may want to check by using e-mail. When a student opens a homework app, before he/she starts to work, it is necessary to enter his/her name, email, and the teacher's email in the corresponding fields. When all of the tasks are completed, the student fills in the last field which refers to the comments on homework, then clicks on "send homework" in order to send the answer to the teacher.

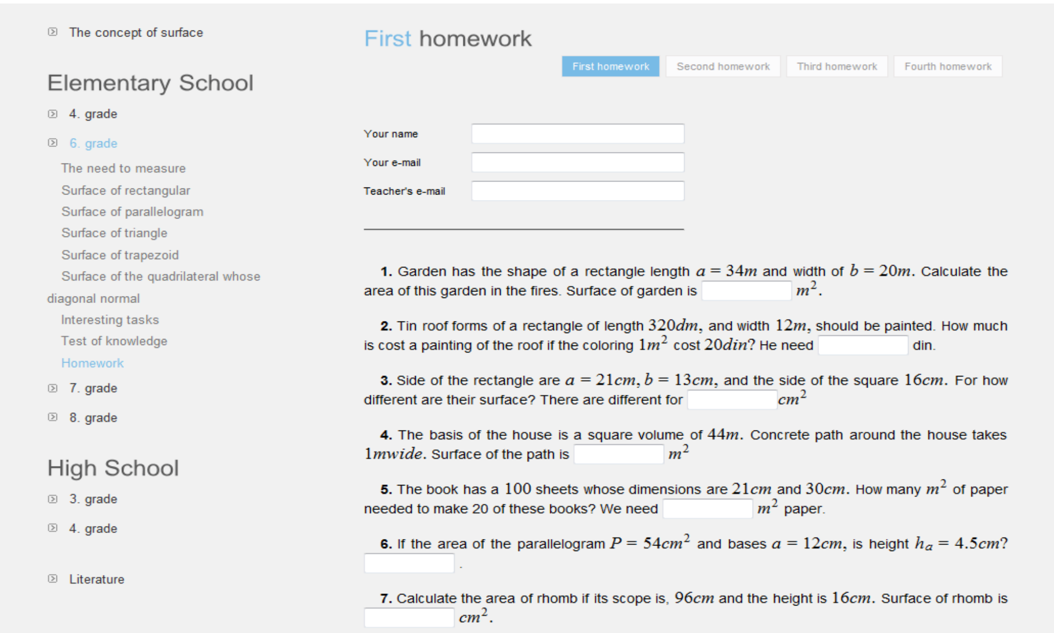

Figure 6. Homework

When a student clicks on "send" button, the student receives a confirmation email (see Figure 7) indicating that the homework has been sent to the teacher.

6 grade of elementary school, First homework. Student <Milos Petricevic> Примљене x

teacher@gmail.com 16.06 (пре 6 минута)

First homework

Notice:
E-mail with your answer is sent to the teacher's e-mail: teacher@gmail.com
Check if the teacher e-mail address is correct. If it is not- send homework again.

Figure 7. Automatic e-mail received by the student

The teacher also gets an email with the homework title, the student's name, an email address, text relating to the tasks, the correct answer(s), and the student's answer(s) in table format (see Figure 8). The teacher needs to review mail and to inform students on the reception of the email, as well as the grade received for the homework tasks.

First homework, 6. grade of elementary school

Student name:	Milos Petricevic		
Student e-mail:	[illegible]		
Text of problem		Corect answer	Student answer
1. Vrt ima oblik pravougaonika dužine a=34m i širine b=20m. Izračunati površinu toga vrta u arima.		680m2	6,8
2. Limeni krov oblika pravougaonika dužine 320dm, a širine 12m, treba obojiti. Koliko košta bojenje tog krova ako se za bojenje 1m2 plaća 20din ?		7680	7680
3. Stranice pravougaonika su a=21cm, b=13cm, a stranica kvadrata je 16cm . Za koliko se razlikuju njihove površine?		17cm2	17
4. Temelj kuće je kvadrat obima 44m . Oko kuće vodi betonska staza širine 1m . Površina staze je ?		48m2	48
5. Sveska ima 100 listova čije su dimenzije 21cm i 30cm. Koliko m2 papira je potrebno da bi se napravilo 20 takvih svezaka?		1260m2 .	126
6. Ako je površina paralelograma P=54cm2 i osnovica a=12cm, da li je visina ha =4,5cm?		Da.	da
7. Izračunati površinu romba ako mu je obim 96cm, a visina 16cm.		384cm2.	65
8. Obim paralelograma je 24cm. Kraća stranica je dva puta manja od duže stranice. Ako je visina koja odgovara dužoj stranici ha =2cm , odrediti dužinu visine koja odgovara kraćoj stranici.		4cm.	3
Comments on homework			

Figure 8. E-mail received by the teacher

3. Conclusion

The main task of teachers is to present a problem to a student, to make it more understandable, and to prepare interactive GeoGebra worksheets that enable students to individually research the concept and to verify new properties of known objects. In this sense, the software package GeoGebra as a tool for modelling and dynamic interactivity, can greatly enhance learning through student discovery activities. In this paper, we have only been able to present a sampling of the many features of GeoGebra and the possibilities that it offers to teachers of mathematics. We have demonstrated a series of interactive educational activities used in elementary and highschools, relating to the term "surface area of figure." Four aspects of a created GeoGebra environment were presented: the learning aspect, explanations, self-testing, and homework extensions. This approach can be applied to any mathematical topic. Teachers can use GeoGebra educational software and present abstract mathematical concepts in a virtual environment wherein students feel very comfortable and confident. In this way, the GeoGebra environment can be viewed as a place where students can learn and play with mathematics at the same time.

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The First Derivative of an Exponential Function with the "White Box/Black Box" Didactical Principle and Observations with GeoGebra

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Abstract. This paper shows how GeoGebra—a dynamic mathematics software—can be used to experiment, visualize and connect various concepts such as function, first derivative, slope, and tangent line. Students were given an assignment to determine the first derivative of the exponential function that they solved while experimenting with GeoGebra. GeoGebra enables students to experiment, model, and research their ideas in order to get desired results for mathematical problems. The software package, GeoGebra, enriches the educational process and opens up new questions and possibilities. Learning and teaching mathematics in contemporary, technology-rich surroundings requires new didactical approaches. The findings in this paper are based on the "White Box/Black Box" didactical principle. Recommendations for mathematics teachers are shared.
Keywords: first derivative; exponential function; dynamic software; GeoGebra; the white box/black box principle

1. Introduction

In traditional mathematical courses, derivatives are introduced through definition. Most of the related classroom time is spent proving and practicing derivation rules. Therefore, there is often not much time for students to actually learn about situations in which derivatives are necessary, or the basic supporting principles. Using GeoGebra, students and teachers are able to explore various mathematical notions in a diverse and profound way. Computer Algebra Systems (CAS), such as GeoGebra, allows the teaching process to concentrate on the modeling process, while complicated calculations are handled by the CAS technology. The use of CAS requires new pedagogical approaches. One such example mentioned in this paper is the didactical principle commonly referred to as the "White Box/Black Box" principle, as developed by Buchberger (Buchberger, 1989) and described in detail in (Heugl, Klinger & Lechner, 1996). The White Box/Black Box principle bridges the traditional teaching of mathematics with the use of available computer technology.

The principle consists of several interlocked stages. The stage when a new problem is solved by hand, or by using already known functions, is called a "White Box" phase. In this phase students distinguish all the basic operations and functions that are necessary to solve the problem. Having the problem carefully analyzed, the students then define a new function that solves the problem in a single step. The problem is therefore solved by a simple function recall (i.e., the press of a button, or entry of a function code) and the related definition of the function is not seen again. This phase is called a "Black Box" phase. It is presumed that the students understand the meaning and the definition of the functions. This process is dynamic. After a certain number of hand calculations or solved examples ("White Box" phase), a function could be used as a "Black Box". One advantage of continuous, by-hand repetition of the function is that students who perhaps did not understand the meaning of a function, or how a certain problem should be solved, still would have a chance to grasp the meaning of the function during repeated practice.

2. The "White Box/Black Box" principle in GeoGebra

In this paper we suggest how students could use educational software such as GeoGebra to visualize and experiment with mathematical facts. We used GeoGebra to carry out the pedagogical observations relating to the "White Box/Black Box" principle within the classroom. GeoGebra was used to handle the computing problems. We followed the didactical pattern of the principle by introducing new concepts with elementary and illustrative methods. For example, we solved some problems of calculus using the limit of the quotient of differences. Therefore our students got a better understanding of the concept of a differential quotient, and of derivatives. The problem of computing the limit was carried out using the computer software.

In this example, it is shown how the properties of tangent line are used to bring students to certain conclusions about the first derivative of the function. Furthermore, this example expounds a distinctive way of performing mathematical tasks. This pedagogical experiment was conducted with 17 senior gymnasium students. This kind of work requires a computer laboratory with Internet access. During the class students worked in pairs, individually or, if it was required, in larger groups.

For a successful classroom computer investigation, a specific pedagogical approach and preparation is required (Tall, 2003). This approach implies technical support such as a well- equipped computer laboratory, and a familiarity with the basics of the software as demonstrated by the teacher and the students involved in the lesson. GeoGebra is one of the leading mathematical educational software packages due to its intuitive and user-friendly interface. GeoGebra is a dynamic mathematics software for all levels of education that joins arithmetic, geometry, algebra, and calculus (Hohenwarter & Jones, 2007). There is a wide range of papers and Internet materials that can help teachers in organizing

GeoGeobra-supported mathematical lessons (www.GeoGebra.org; www.GeoGebra.org/en/wiki). A well-prepared plan of the lesson, including the prediction of common as well as unexpected student questions, improves the teaching process. It is a great advantage if the files containing the lessons are prepared ahead of time with lots of tasks, assignments, and questions for the students to explore. Furthermore, GeoGebra is advantageous due to its visual features that can motivate students to explore, and can make a lesson more interesting and dynamic (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008).

3. The Task and GeoGebra Solution

As we have noted above, the "White Box" phase requires a theoretical knowledge. For example, all of our students were familiar with the following definitions:

$$tg\varphi = \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = \frac{\Delta y}{\Delta x} = f'(x_0) \text{, slope of the tangent line} \quad (1)$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)\text{, definition of derivative} \quad (2)$$

This theoretical knowledge was considered as part of the "White box".

The "Black Box" phase requires an upgrading of students' existing theoretical knowledge through the use of modeling and application. The students were set the task to find the derivative of the function $y = a^{x}$. It was considered as a "Black Box". The first derivative is an equation for the slope of a tangent line to a curve at an indicated point. The first derivative may be found using a derivative definition (1). With the purpose of making the task of finding the first derivative easier and more obvious, students observed specific function $y = 2^x$. Using the definition (1) students obtained the following:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2^{x+h}-2^x}{h} = \lim_{h\to 0}\frac{2^x(2^h-1)}{h} = 2^x\lim_{h\to 0}\frac{2^h-1}{h} = f'(x) \quad (3)$$

Using the textbook, the task would be solved in the previous steps, only it would be concluded that the value of the limit $\lim_{h\to 0}\frac{2^h-1}{h}$ was known, as this limit is considered as an important limit. Instead of this more traditional procedure, the use of GeoGebra offers an approach that includes a wide range of experimentation, independent reasoning, and the use of relevant technology.

When using GeoGebra, after analyzing result (3), students concluded that the result of differentiation is a function, but the question at that point was, Which particular function should be expected? With the aim of obtaining the resulting function, students used GeoGebra software to explore various possibilities and experiment with the initial conditions. Plotting a graph in GeoGebra was helpful when trying to visualize the tangent line. The steps described in the text were then used to attain the correct solution.

3.1 The First GeoGebra Worksheet

This part of the task was the same for all students and they worked in pairs. The function $y = 2^x$ was keyed into the input field by the students. Subsequently, the slider marked with *h* was added to the GeoGebra sheet. The slider *h* is considered as a very small value, almost zero, being set between 0 and 0.1. These facts were adjusted to the GeoGebra features and entered as the function $g(x) = \frac{2^h - 1}{h}$. Further on, these steps were animated using the GeoGebra features *Navigation Bar for Constructing Steps* and the *Play Button*. Playing the steps, students noticed that $\lim_{h \to 0} \frac{2^h - 1}{h}$ was constant and valued around 0.7. They marked that belt in colour, in order to make this fact more evident (see Figure 1).

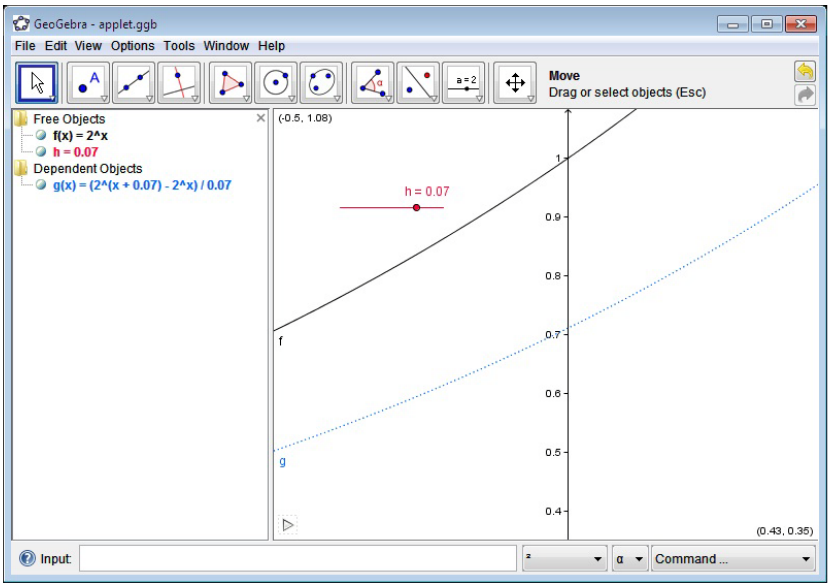

Figure 1. Visualization of the first derivative of the function $y = 2^x$

The students' task was to determine the specific constant. Note that this part could also be presented as a previously-prepared GeoGebra applet which could gradually introduce students to the topic, and thereby save class time. With the applet application, students could analyze its features with the teacher.

3.2 The Second GeoGebra Worksheet

This part of solving the task was done strictly on an individual basis. Each student was assigned a different function. For example, one took the function $y = 3^x$, another $y = 5^x$, and another $y = \left(\frac{1}{2}\right)^x$ etc. The idea was that each student chose a different base for the exponential function $y = a^x$ in order to experiment with lots of different functions and to ultimately arrive at the correct solution. The steps that every student followed are described in the following text. A student's solution is shown in Figure 2.

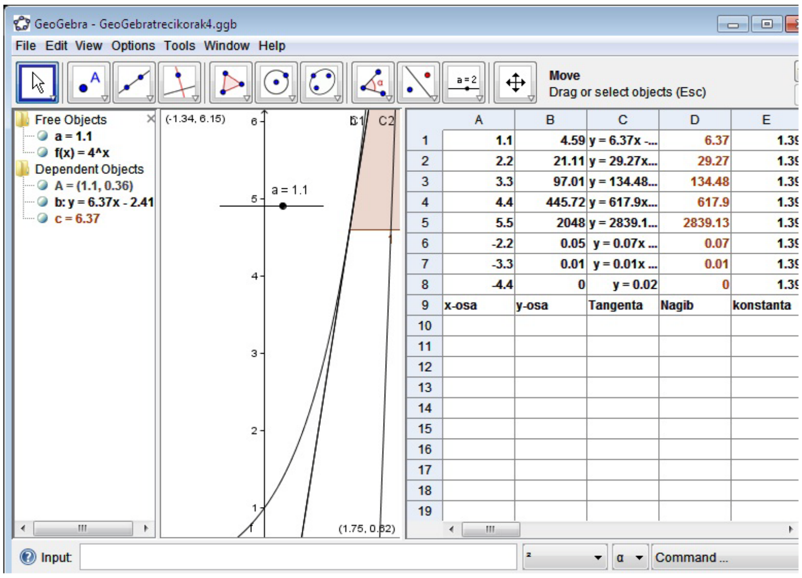

Figure 2. GeoGebra worksheet with function $y = a^x$, regarding its base, typed by student

The introduction of point A on the graph of the function $y = a^x$ followed. It was defined with a slider. The slider option was chosen in order to increase the number of different points that should provide opportunities for students to reason inductively. The next step was introducing line *b,* that represented the tangent line of the function $y=a^x$ at the point *A.* A *tangent line* is a line that locally touches a curve at one, and only one, point and this is an embedded function within the GeoGebra software. Furthermore, students used another embedded GeoGebra feature known as the slope of the function. The students' theoretical knowledge was limited to the definition of the derivative of a function and its connection to the tangent line at the point of the function graph. After revising the theoretical framework students concluded the following:

$$a^x \lim_{h \to 0} \frac{a^h - 1}{h} = f'(x) \tag{4}$$

$$\lim_{h \to 0} \frac{a^h - 1}{h} = \frac{f'(x)}{a^x} \tag{5}$$

Students noted these results in a table, which is another very useful feature in GeoGebra. In the table they noted the following results:

In column *A*, students inputted approximately ten x-axis values for points of the graph of the function. Their x- axis values were defined as x_A, $2x_A$, $3x_A$, etc., where the value x_A was defined by the slider. In column *B*, students defined y-axis values of the point. Column *C* was reserved for the equation of the tangent line at the determined point, and column *D* contained the appropriate slope's values. Finally, column F contained the results of the ratio,

$$\frac{column\ \ D}{column\ \ B} \tag{6}$$

corresponding to the previously analyzed and obtained

$$\frac{f'(x)}{a^x} \qquad (7)$$

The final result was the constant number in each field of column F.

3.3 The Third GeoGebra Worksheet

This part required collaborative group work during which time students discussed and analyzed the obtained data. Students noticed that the limit $\lim_{h\to 0}\frac{a^h-1}{h}$ was constant and also observed that even though the points of the graphed function changed, the constant remained the same. Every student had a different function, and so they all obtained different constants. They collected all constants, overall 17 constants, in one GeoGebra file (see Figure 3).

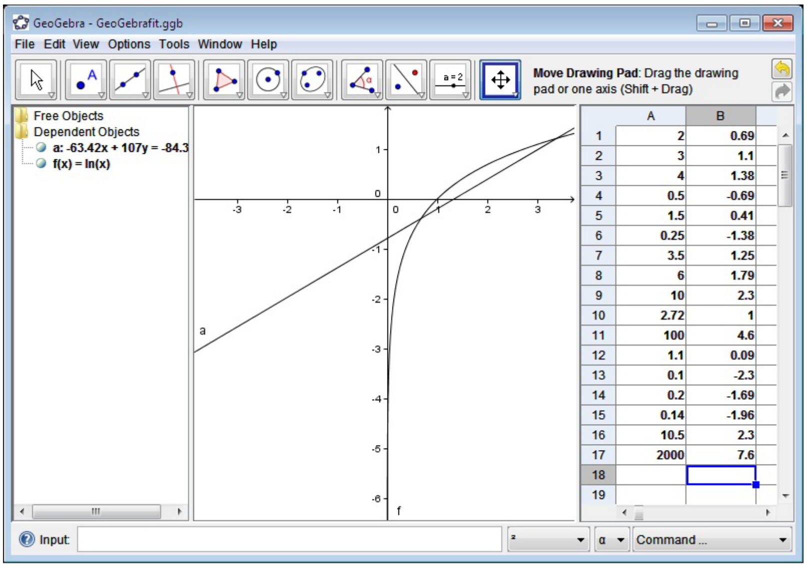

Figure 3. The students' final conclusion

They used the *Table Option* and then another GeoGebra feature for fitting the data. The first column in the table represented values of the base of the function, and the second column represented the obtained constant. They experimented with the features *FitLin, FitExp, FitLog* and came to the conclusion that the best fitting curve was $y = \ln a$. Finally, the conclusion was that the derivative of the function was $f'(x) = (a^x)' = a^x \ln a$.

4. Conclusion

While learning calculus, many students might have difficulties grasping the concept of the derivative as a function that outputs the value of the slope of a tangent. Having this in mind, we tried to get students more actively and visually involved in the development of the derivative. That goal was largely accomplished by using GeoGebra and the "White Box/Black Box" principle during the teaching process. Accordingly, using GeoGebra in the classroom provides a higher quality of teaching, helps the students with tedious computation, and allows visualization of the problems and pattern recognition for students.

Also, the use of GeoGebra supports students in making new conjectures and in tackling experimentation. In this example, students not only proved and solved the given task, but they also revised and connected several very important mathematical facts. The possibility of recovering mathematical content experimentally is very motivating for many students. The use of a computer gives many opportunities for experiments. The usage of computers in the classroom should be followed by adequate didactical principles, for example the "White Box/Black Box" principle. Although investigations like the one described herein can be quite time-consuming, leading some students and teachers to prefer more traditional methods, its long-term benefits are nonetheless quite remarkable.

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Modeling and Visualization Process of the Curve of Pen Point by GeoGebra

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Abstract. This study describes the mathematical construction of a real-life model by means of parametric equations, as well as the two- and three-dimensional visualization of the model using the software GeoGebra. The model was initially considered as "determining the parametric equation of the curve formed on a plane by the point of a pen, positioned on an obstacle of height h, during the process of raising the pen vertically to the surface by linearly moving its back-end on the surface." Firstly a solution was sought for this problem in two dimensions. Based on this problem, two additional sub-problems were formed on a plane, and parametric equations were calculated for these sub-problems as well. The curves formed by these parametric equations were then visualized using GeoGebra. In the second stage, the model was improved, and the parametric equation of the curve formed in the space by the pen point as a result of moving the pen's back-end along any function was determined. The curve formed by this parametric equation was also visualized using the GeoGebra 3-D environment. It is expected that determining mathematical concepts and relationships based on real-life models with these types of training tasks, as well as jointly considering the algebraic and geometric representations during the process, will improve the students' perceptions relating to mathematics.
Keywords: modeling; GeoGebra; parametric equation; real-life problems; 3-dimensional modeling

1. Introduction

Mankind has devised and continually developed mathematics due to the necessity of making certain calculations in daily life. Hans Freudenthal suggested that, historically, mathematics found its origins in real-life problems, that aspects of real life were then mathematized, and that formal mathematical information was achieved afterwards. Hans Freudenthal has named this approach the Realistic Mathematics Education (RME) (Altun, 2008). This approach encompasses two main concepts, which are the horizontal mathematization, and the vertical mathematization. Horizontal mathematization involves the mathematical expression of real-life problems in a mathematical sense. In other words, it is the mathematization of real-life models. Vertical mathematization, on the other hand, involves the re-expression of mathematics with the use of symbols (Freudenthal, 1991). In this context, horizontal mathematization makes use of models, graphs and diagrams (Freudenthal, 1991; Streefland, 1991).

A sub-dimension of the RME is modeling (Streefland, 1991). Modeling is the process of creating a model for a problematic situation. In this respect, the "model" refers to a product formed at the end of a process, while "modeling" refers to the process of creating a physical, symbolic, or abstract model for a particular situation (Kertil, 2008). Modeling activities that are performed for problematic situations actually provide mathematics teachers the opportunity for self-development (Lesh & Doerr, 2008).

The mathematical modeling of a real-life problem with Computer Algebra Systems (CAS) and Dynamic Geometry Software (DGS) is considered by researchers focusing on this field as a problem-solving activity that suits the purposes of mathematical learning. In fact, Zbiek and Conner (2006) have indicated that modeling specifically contributes to the understanding of known mathematical concepts, to the learning of new mathematical concepts, to establishing interdisciplinary relationships, and to both the conceptual and procedural development of students through the detailed demonstration of the applicability of mathematical concepts in real-life.

There are various studies demonstrating the importance of dynamic geometry software and computer algebra systems as tools for realistic mathematics education (Aktümen, 2013; Aktümen, Baltaci, & Yildiz, 2011; Aktümen & Kabaca, 2012; Kabaca & Aktümen, 2010; Widjaja & Heck, 2003). In addition to these, numerous studies have been performed on real-life problems in mathematics education (Aydin & Monoghan; Aydin-Unal & Ipek, 2009; Fauzan, Slettenhaar, & Plomp, 2002; Kwon, 2002; Oldknow & Taylor, 2008; Van Den Heuvel-Panhuizen, 2000). Furthermore, there are a gradually increasing number of studies investigating real-life problems with DGS (Aktümen & Kabaca, 2012; Gecü & Özdener, 2010; Gittinger, 2012; Kabaca & Aktümen, 2010; Widjaja & Heck, 2003). At the same time, there are also studies in the literature regarding three-dimensional modeling with DGS (Aktümen, 2013; Aktümen, Baltaci, & Yildiz, 2011; Aktümen, Doruk, & Kabaca, 2012; Oldknow, 2009; Oldknow & Tetlow, 2008).

In recent times, it can be seen that parametric equations are also being used when performing modeling studies with DGS (Aktümen, 2013; Aktümen & Kabaca, 2012; Filler, 2012). We can see many reflections of the concept of parametric equations in daily life. For example, in the manufacture of Computerized Numerical Control (CNC) milling machines, the necessary calculations are performed by using parametric equations (Özel & Inan, 2001). In this study, the GeoGebra version 5.0 Beta has been chosen for the modeling of real-life problems by using parametric equations, as it has the same features as dynamic geometry software and computer algebra systems, and allows for the use of three-dimensional modeling. These modeling processes were developed by solving the following four problems.

1) The three problems for the *x*-*y* axis are specified below.
    a. What is the parametric equation of the curve formed on a plane by the point of a pen, positioned on an obstacle of height *h*, during the process of raising the pen vertically to the surface by linearly moving its back-end on the surface.
    b. By moving the back-end of a pen linearly on a surface, whose front-end is affixed to a ring at a certain height, what is the form and parametric equation of the curve formed by the pen point?
    c. For the angle $(0,\pi)$ that a pen whose front-end (or extension) is affixed to a ring at a certain height forms with the x axis, what is the form and parametric equation of the curve formed by the pen point?
2) The problem for which an answer would be investigated in space was: "By moving on any function the back-end of a pen whose front-end (or extension) is affixed to a ring at a certain height, what is the form and parametric equation of the curve formed by the pen point?

2. Research Methods

In this study*, the modeling process was evaluated separately on both the plane and the space by using a model that had been developed based on real-life. The first three problems were modeled on a plane, while the last problem was modeled to in a space. The modeling and GeoGebra visualization processes for each one of these problems are provided below.

2.1 Modeling Process for an Option of the Problem 1

For the pen which has an obstacle of height *h* in its front, and for which the distance between its back-end to the origin is *a* units, the situation prior to its movement is provided in Figure 1.The back-end of the pen is positioned on point O, while its point (front-end) is positioned on point B.

* Note: A part of this study was presented in the Ninth Mathematics Symposium as a poster presentation on October 20, 2010.

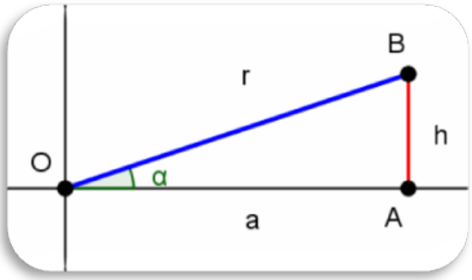


Figure 1. Two-dimensional model of a pen with an obstacle of height *h* in front

According to Figure 1, the first situation is reflected by $r^2 = a^2 + h^2 \rightarrow r = \sqrt{a^2 + h^2}$. The situation resulting from a certain amount of linear movement of the back-end of the pen is provided in Figure 2.

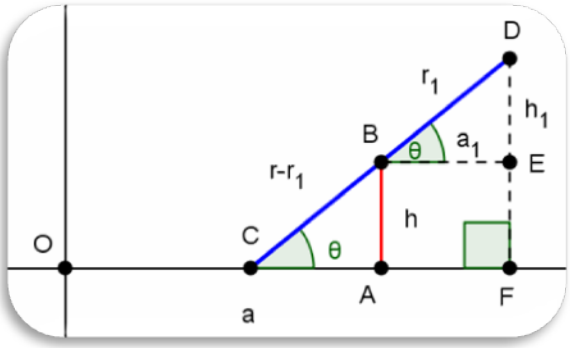


Figure 2. Situation resulting from the linear movement of the back-end of the pen

When the *x* and *y* coordinates of point D are determined accordingly, we obtain:

$$x = a + a_1 \text{ and } y = h + h_1$$

By utilizing triangle BED, we obtain:

$$\cos\theta = \frac{a_1}{r_1},\ \sin\theta = \frac{h_1}{r_1}$$

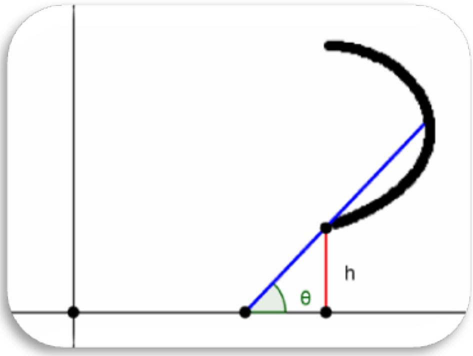
$$a_1 = r_1 \cos\theta,\ h_1 = r_1 \sin\theta$$

As a result:

$$x = a + r_1 \cos\theta \text{ and } y = h + r_1 \sin\theta \qquad (1)$$

Now, the parametric equation will be obtained when the value of r_1 is calculated. For triangle BCA, $\sin\theta = \frac{h}{r - r_1}$. Thus, $r_1 = r - \frac{h}{\sin\theta}$. In this case, $r_1 = \sqrt{a^2 + h^2} - \frac{h}{\sin\theta}$. When this value is inserted into the expression provided in (1), the coordinates of D then become $x = a + \left[\sqrt{a^2 + h^2} - \frac{h}{\sin\theta}\right]\cos\theta$ and $y = h + \left[\sqrt{a^2 + h^2} - \frac{h}{\sin\theta}\right]\sin\theta$.

Thus, the parametric equation of the curve formed on a plane by the point of a pen, positioned on an obstacle of height *h*, during the process of raising the pen vertically to the surface by linearly moving its back-end on the surface (Figure 3) is given by:

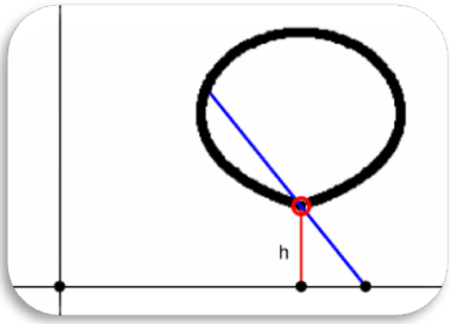


$$x(\theta) = a + \left(\sqrt{a^2 + h^2} - \frac{h}{\sin(\theta)}\right)\cos(\theta)$$
$$y(\theta) = h + \left(\sqrt{a^2 + h^2} - \frac{h}{\sin(\theta)}\right)\sin(\theta)$$
$$\text{with } \theta \in \left[\arctan(\frac{h}{a}), \frac{\pi}{2}\right]$$

Figure 3. Visualization of the parametric equation for the first problem

2.2 Modeling Process for "b" Option of Problem 1

As a result of the linear movement on the surface of the back-end of a pen affixed to a ring, the angle formed with the *x*-axis assumes values that fall between $\left[\arctan(\frac{h}{a}), \pi - \arctan(\frac{h}{a})\right]$. The parametric equation of the curve formed by the point of the pen (Figure 4) is given by:

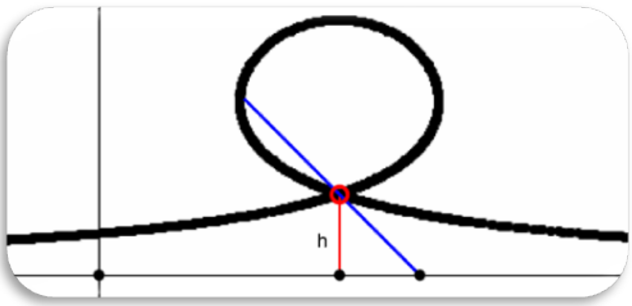


$$x(\theta) = a + \left(\sqrt{a^2 + h^2} - \frac{h}{\sin(\theta)}\right)\cos(\theta)$$
$$y(\theta) = h + \left(\sqrt{a^2 + h^2} - \frac{h}{\sin(\theta)}\right)\sin(\theta)$$
$$\text{with } \theta \in \left[\arctan(\frac{h}{a}), \pi - \arctan(\frac{h}{a})\right]$$

Figure 4. The modeling and parametric equation for the second situation

2.3 Modeling Process for "c" Option of Problem 1:

By linearly moving on the surface the back-end of the pen affixed to a ring such that its angle with the *x*-axis falls within the $(0, \pi)$ range, the parametric equation of the curve formed by the pen point (Figure 5) becomes:



$$x(\theta) = a + \left(\sqrt{a^2 + h^2} - \frac{h}{\sin(\theta)}\right)\cos(\theta)$$
$$y(\theta) = h + \left(\sqrt{a^2 + h^2} - \frac{h}{\sin(\theta)}\right)\sin(\theta)$$
$$\text{with } \theta \in (0, \pi)$$

Figure 5. The modeling and parametric equation for the third situation

2.4 Modeling Process for Problem 2

Figure 6 provides the model for the problem: "By moving on any function the back-end of a pen whose front-end (or extension) is affixed to a ring at a certain height, what is the form and parametric equation of the curve formed by the pen point?"

Figure 6. Three-dimensional description of the model

For resolution of this problem, the labeling described in Figure 7 was employed.

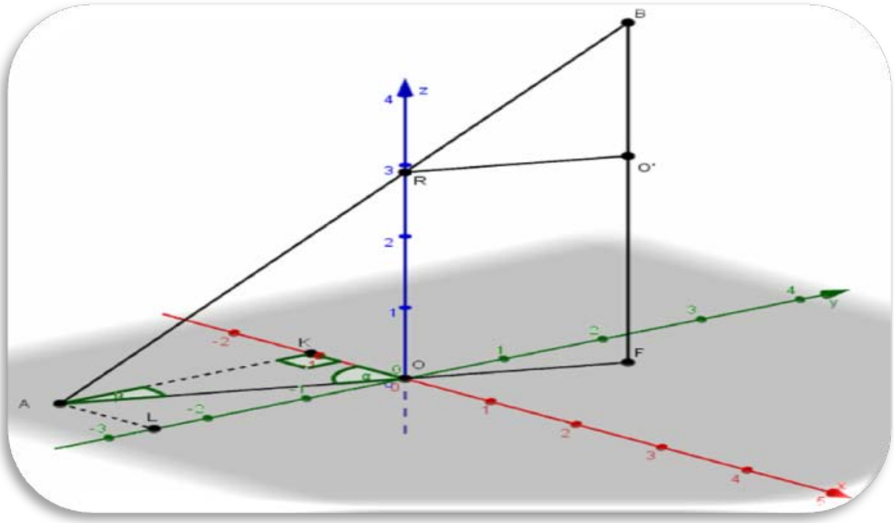


Figure 7. The Formation of the model in three dimensions

With points A, O, F, R, O' and B being planar, $\angle KOA = \alpha$, $\angle KAO = \beta$, $\angle OAR = \theta$, $|OR| = h$, $|AB| = k$, $|OK| = t$, $|OL| = f(t)$ (with $f(x)$ being a function determined by the user), and the line segment AB representing the pen;

$|AO| = \sqrt{t^2 + f(t)^2}$, since $m\angle OKA = 90^\circ$.

Since $m\angle AOR = 90^\circ$, $|AR| = \sqrt{t^2 + f(t)^2 + h^2}$, thus $|RB| = k - \sqrt{t^2 + f(t)^2 + h^2}$.

Since $m\angle OAR = m\angle O'RB$, $|RO'| = |RB|.\cos\theta = \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right).\cos\theta$.

And since ΔARO ≅ ΔRBO' and $\cos\theta = \dfrac{\sqrt{t^2 + f(t)^2}}{\sqrt{t^2 + f(t)^2 + h^2}}$,

$$|RO'| = \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right).\frac{\sqrt{t^2 + f(t)^2}}{\sqrt{t^2 + f(t)^2 + h^2}}.$$

Let us now determine the coordinates of point B.

Since $|RO'| = |OF|$, $B = (OF.\cos\alpha, OF.\sin\alpha, k.\sin\theta)$.

It is calculated that $\cos\alpha = \dfrac{-t}{\sqrt{t^2 + f(t)^2}}$, $\sin\alpha = \dfrac{-f(t)}{\sqrt{t^2 + f(t)^2}}$ and $\sin\theta = \dfrac{h}{\sqrt{t^2 + f(t)^2 + h^2}}$

Since $|OF| = \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right)\dfrac{\sqrt{t^2 + f(t)^2}}{\sqrt{t^2 + f(t)^2 + h^2}}$, the *x*, *y* and *z* coordinates of point B are determined as:

$$x(B) = \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right)\frac{\sqrt{t^2 + f(t)^2}}{\sqrt{t^2 + f(t)^2 + h^2}}.\frac{-t}{\sqrt{t^2 + f(t)^2}}$$

$$= \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right)\frac{-t}{\sqrt{t^2 + f(t)^2 + h^2}} = \frac{\left(-kt + t\sqrt{t^2 + f(t)^2 + h^2}\right)}{\sqrt{t^2 + f(t)^2 + h^2}}$$

$$= t\left(1 - \frac{k}{\sqrt{t^2 + f(t)^2 + h^2}}\right)$$

$$y(B) = \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right)\frac{\sqrt{t^2 + f(t)^2}}{\sqrt{t^2 + f(t)^2 + h^2}}.\frac{-f(t)}{\sqrt{t^2 + f(t)^2}}$$

$$= \left(k - \sqrt{t^2 + f(t)^2 + h^2}\right)\frac{-f(t)}{\sqrt{t^2 + f(t)^2 + h^2}} = \frac{\left(-kf(t) + f(t)\sqrt{t^2 + f(t)^2 + h^2}\right)}{\sqrt{t^2 + f(t)^2 + h^2}}$$

$$= f(t)\left(1 - \frac{k}{\sqrt{t^2 + f(t)^2 + h^2}}\right)$$

$$z(B) = k\frac{h}{\sqrt{t^2 + f(t)^2 + h^2}}$$

Provided below are the curves formed by the point of the pen for certain functions in which its back-end is moved.

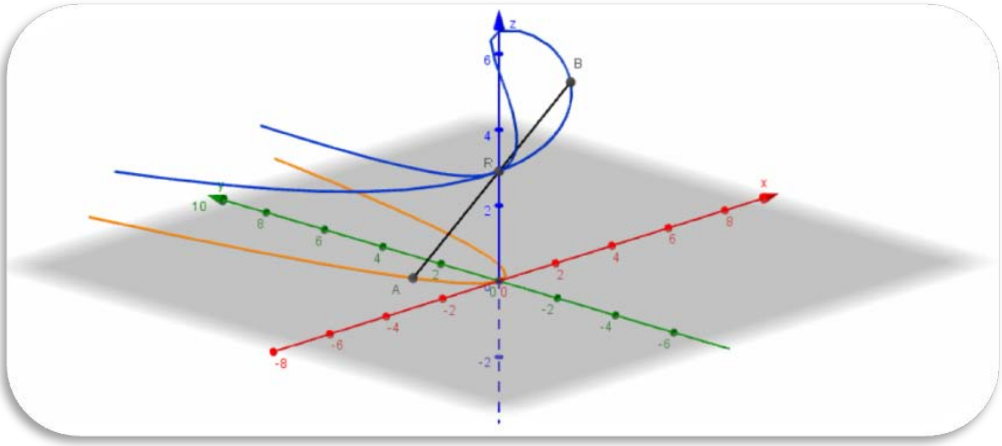


Figure 8. Curve formed as a result of the movement of the back-end of the pen on the $f(x)=x^2$ function

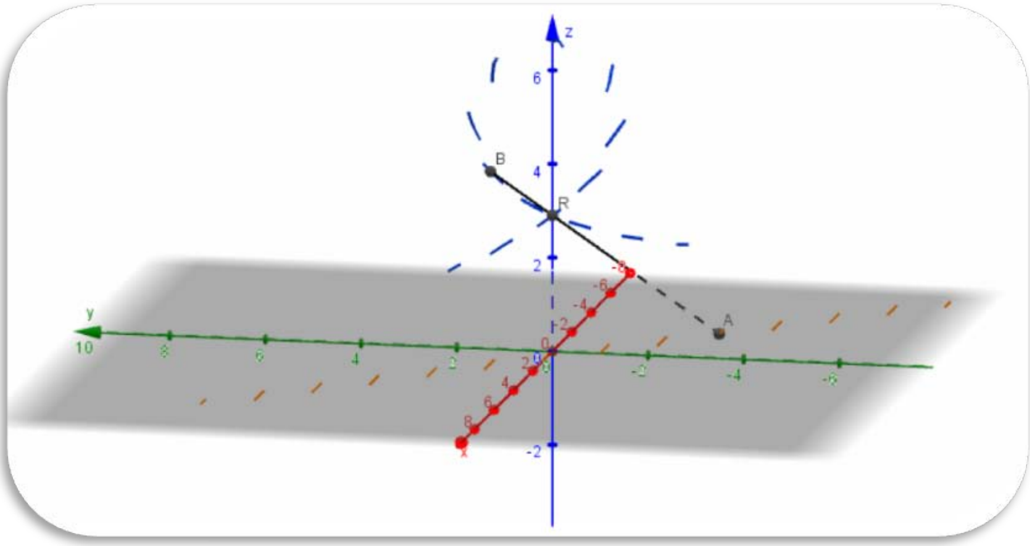


Figure 9. Curve formed as a result of the movement of the back-end of the pen on the $f(x)=\|x\|$ function

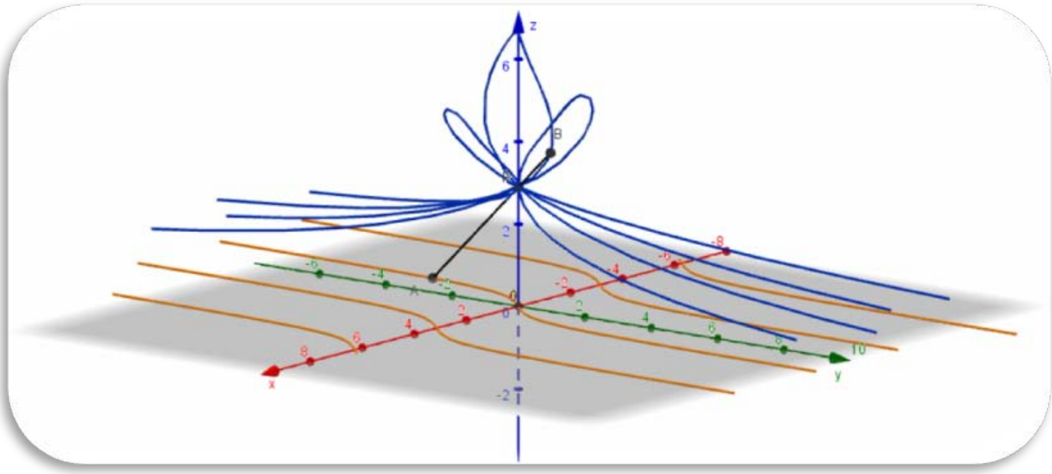


Figure 10. Curve formed as a result of the movement of the back-end of the pen on the $f(x)=\tan x$ function

3. Conclusions

In this study, the modeling processes for real-life problems were described by forming problem situations based on a real-life model. At the end of these processes, the parametric equations of the curve created by the pen point curve were formulated both for the plane and the space. Visualization of these processes was ensured by using the GeoGebra 5.0 Beta software, which is dynamic geometry software. It is expected that determining mathematical concepts and relationships based on real-life models with these types of training tasks, and jointly considering the algebraic and geometric representations during the process, will improve the overall students' perceptions of mathematics. In fact, Freudenthal has expressed that it is necessary to associate learning in mathematics classes with real-life, and that sustaining this approach would be one of the most suitable methods to follow (Gravemeijer & Terwel, 2000; Muijs & Reynolds, 2011; Wubbels, Korthagen, & Broekman, 1997). It can thus be argued that a dynamic model pertaining to a real-life problem can assist us in explaining and interpreting mathematical models, and thereby support a better understanding of a mathematical model by demonstrating its graphical representation and relationships (Doerr & Pratt, 2008; Duval, 1999). We are suggesting that the GeoGebra 5.0 Beta software can provide a suitable environment for designing such models, and that by using this software students become more engaged in their mathematics learning. We also contend that developing such models can assist students who have difficulties thinking in 3-dimensions in terms of developing their spatial skills.

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GeoGebra Software Use within a Content and Language Integrated Learning Environment

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Abstract. This paper presents results of a research study focusing on the analysis, comparison, and description of students' attitudes towards the teaching of mathematics lessons presented in a foreign language (English) using the Content and Language Integrated Learning (CLIL) method in three elementary schools. It also highlights the difference between the attitudes of the CLIL method learners and those of their student counterparts who experienced similar mathematics lessons but in their mother tongue (Czech). The aim of the research is to test the hypothesis that the teaching of mathematics in a foreign language by the CLIL method would be evaluated positively by participating students. The research also focused on the question of whether or not, or to what degree, the implementation of the foreign language (English) along with the use of an interactive tool, such as GeoGebra software in mathematics lessons, was perceived as being meaningful and as significantly improving the effectiveness of student learning.
Keywords: CLIL method; computer assisted learning; GeoGebra; math teaching

1. Introduction

It is generally acknowledged (Ellis, 2002; Gay, 1988) that foreign (second) languages are most effectively learned in a context that is meaningful and interesting for the learners. This key fact makes language teachers and methodologists think carefully about the importance of the context in terms of a perceived sense of authenticity or real-life implications within the learning process. The effectiveness of the usage of a second language as a medium to convey informational content of interest and relevance to the learners has become a rationale underlying content-based, second language learning and teaching. Nowadays, the knowledge of foreign languages within contemporary society is commonly held in high esteem. There are many ways and possibilities to support foreign language teaching and learning, but one of them which is used more and more often at lower and upper secondary and at the university level is the Content and Language Integrated Learning (CLIL) method.

2. Content and Language Integrated Learning Method

The Content and Language Integrated Learning (CLIL) method, coined in 1994 by David Marsh (1994), was further promoted by the European Union as an umbrella term to denote any classroom situation in which content and language are taught simultaneously. Amongst teachers, it is most common just to refer to this method as bilingual teaching. According to Marsh (2002), CLIL refers to situations where subjects, or parts of subjects, are taught through a foreign (target) language with dual-focused aims, namely "the learning of content, and the simultaneous learning of a foreign language" (p. 2). As the learning is simultaneous, students are exposed to the target languages without requiring extra time in the curriculum.

David Graddol (2006) described CLIL as,

> [A]n approach to bilingual education in which both curriculum content (such as science or geography) and English are taught together. It differs from simple English-medium education in that the learner is not necessarily expected to have the English proficiency required to cope with the subject before beginning study. (p. 86)

As CLIL teaching focuses on both the content and the target language, it therefore requires collaboration between subject teachers and the language teachers, as well as new kinds of pedagogical practices and interactive approaches to be used with carefully designed learning tasks.

3. ICT in Teaching Mathematics and Languages

Technology offers teachers many opportunities to use multiple learning tools in the teaching and learning process, and the use of computers in education brings numerous benefits. In teaching mathematics, technology helps to visualise processes that cannot be done only with a piece of chalk on the blackboard. Dynamic and interactive mathematics learning environments have been developed to support the learning of mathematics through free exploration in a less constrained context. Certain software packages have interactivity and dynamism as key affordances. These are, according to Martinovic and Karadag (2012), especially suitable for enhancing learning and teaching with the technology of the essentially dynamic mathematics concepts.

In language teaching, computers have been used for more then 30 years. Nowadays, technological and pedagogical developments allow teachers to better integrate computer technology into language and mathematics learning processes. Multimedia programs incorporating speech-recognition software can immerse students into rich environments for language practice. Various mathematics and language software titles can create suitable environments for individual work and student self-study. The development of the Internet has allowed students to investigate language use in an authentic, multimedia context, to communicate in the target language, to access textual and multimedia information, and to publish for a global audience.

But, how do we use computers effectively and not divert students from the essence of the subject matter? This is explored in many studies and research works. We can name, for example, the works of Kutzler (2003) or Healy and Sutherland's work (1990). This is closely connected with a sophisticated use of computers which can lead to even more complex questions: Is it possible to reach the situation where computer technologies would, generally speaking, enable students' individual exploration, and would it be beneficial in the development of their creativity and invention? Can computer technologies help with the actual creation of a conceptual structure in learning? (See, for example, the works of Balacheff & Kaput, 1996; Balacheff et al., 2006; Binterová & Fuchs, 2010).

The development of psychological aspects of learning resulted in applications of new teaching methods and approaches that were designed to respect the personalities of people and their individual cognitive styles, self-knowledge, and cognitive situations. These factors should be taken into account in the use of modern technologies in the teaching and learning process. This is investigated, for example, in studies done by Pitta and Christou (2008) who focused on the influence of dynamic geometry on the cognitive styles of students. Davis and Simmt (2003) verified that learning in a classroom represents a complex reality and the implementation of technologies into the learning process could bring even more complexities.

All of this is closely connected with the necessity to define new representations and ways of mathematical modelling in a computer-assisted teaching process that influences the mathematical thinking of students. In connection with the teaching environment, Papert (1998) explained that the computer-assisted teaching environment is a "micro-world," a place where computers can have an impact on children who are being provided with meaningful experiences. In this environment, children have free access to, and control of their own learning experiences, children and teachers learn together, teachers encourage peer tutoring, and students can increase their knowledge and develop their mathematical thinking. Edwards (1995) emphasised how that the micro-world experiences are built on the pre-exsiting knowledge of students. Effective teachers provide their students with technology-rich spaces wherein they can explore and realize their ideas, verify mathematical theorems, and also check their own hypotheses (Mousoulides & Philippou, 2004).

Information and communication technologies may have a facilitative impact on concept creation, and may therefore positively influence the achievement of learning goals. To utilise new technologies for the creation of a good learning environment for students in mathematics and the teaching methodology of mathematics, it is necessary to secure relevant competencies for teachers. In education, it is not always true that the utilisation of modern technologies in lessons equals effectiveness. Rather, the utilisation of modern technologies in accordance with the right teaching principles in lessons can increase the probability of effective learning.

The results of the research investigations entitled, *Interactive Whiteboards, Pedagogy and Pupil Performance Evaluation: An Evaluation of Schools' Whiteboard Expansion* (Moss et al., 2007), show that teachers do not, at the moment, possess expertise in creating well-balanced and methodologically correct teaching materials. Their materials lack comprehensibility and simplicity for use in individual work. As information and communication technologies may have a positive impact on the teaching of mathematics and the learning processes of students, many countries have implemented within their educational curricula a requirement for the maximum utilisation of technology (Hennessy, Ruthven, & Brindley, 2005). This implementation is not easy, as there are many influencing factors at play. These factors are, for instance, the attitude of teachers towards the implementation of new technologies, their concerns about the innovative process, the readiness and further education of teachers, and the choice of suitable programmes and interactive materials for successful implementation (Hennessy et al., 2005). Gibson (2001) says that technology itself cannot really change anything.

4. Students' Perceptions of the Educational Climate

Why is this research focusing on educational climate? It is important to realise that the classroom is a social environment in which students spend many hours during their schooling, and so gain many experiences. For this reason, the social quality in the classroom is important for forming students' feelings, and their attitudes to their peers, to their teachers, to the school subjects, and also to education itself. The time spent in school is a way of learning and of acquiring social experiences which serve as a basis for character development, attitudes towards life-long learning, and future employment in society (Lambert & McCombs, 1998; Lambert, Abbott-Shim, & McCarty, 2002). A necessary, but not solely sufficient, condition for learning is to create the right social climate. This positive climate is formed by mutual relationships between the student and their teacher, and between fellow students, and is derived from a combination of the quality of these relationships, student motivation, and student performance (Fraser, 1986).

The aim of this paper is not a complete definition of all of the relevant terminology. However, it is necessary to define educational climate, as it is understood, perceived, and presented in this research. Mareš (2000) understands school climate as a "set of ways of perception, experiences, evaluation and reactions of all school participants to what has taken place, is taking place and is to take place in the school environment" (p. 242). This definition is a starting point for our definition of educational climate, which we characterize as a social phenomenon. From the content point of view, it includes the perceptions and experiences of students and their evaluation of the situations happening during the teaching and learning process. We understand the educational climate during lessons of mathematics as a certain quality of the school environment, the life-space of students formed during mathematics teaching and learning. Students perceive these processes in their own way, and they react to them and can characterize them together with the environment wherein they experience the processes.

The adjective "social" is linked with the characteristics of the psycho-social environment in educational establishments. Empirical studies and international comparisons show that there are significant differences between schools regarding their social climate (Allodi, 2007). Research has verified that school climate significantly influences students' school achievement and even their employment after graduating from school (Rutter, 2000). Fraser and Tobin (1991) stated that classroom climate may be influenced by students' behaviour, the level of knowledge in the school, students' study results, their motivation in cognitive fields, and their attitudes to education itself. If the classroom climate is unfriendly, then anxiety, restlessness, and scepticism may appear, leading to intellectual and cognitive depression. On the other hand, in classrooms where the climate is positive and friendly, there is evidence that the abilities and achievements of students are higher (Grecmanová, 2008, p. 63).

5. Research Purpose

The research presented in this paper was conducted during the years 2009-2011 in three elementary schools in mathematics lessons presented in the English language using the CLIL method, within a project supported by the European Union called *Interconnection of a Foreign Language and a Content School Subject in Elementary School.* The project followed, in the authors' opinion, the successful experimental teaching of mathematics in English in one of the participating schools in October 2006 (Šulista, 2012). The experimental teaching was designed to explore how the implementation of the CLIL method in mathematics lessons was possible, and if it would be well-accepted by students. The evaluation of the experimental teaching came with the following opinions of 49 students:

- 62% of the students stated that they liked the classes of Mathematics in English;
- 53% of the students thought that they had understood everything involved in the lesson concerning English;

- 16% of the students thought that they had understood everything involved in the lesson concerning mathematics;
- 24% of the students would like to continue with mathematics in English;
- 20% of the students would like to have more subjects in English; and
- 73% of the students thought that they had learnt many interesting and useful words and phrases apart from mathematical ones.

This experimental implementation showed that the majority of the participating students liked mathematics lessons in English. The analysis of study results made after experimental teaching in both the mother tongue (Czech) and the target language (English) also showed that the target language does not have a negative impact on the students' acquired knowledge of mathematics. The students who received teaching in the target language did not achieve statistically worse results than their counterparts who received teaching in their mother tongue. However, the experimental teaching results do not imply anything about the educational climate.

The research sought to explore the perceptions of students regarding the two types of mathematical learning environments. Our research design, which was both quantitative and qualitative in nature, focused on the difference in students' learning in classes during an implementation of the CLIL method, supported with an interactive environment in mathematics lessons, and the learning of their counterparts during traditional mathematics lessons presented in their mother tongue, at the lower-secondary educational level.

The aim of the research study was to analyse, compare, and describe educational climate in mathematics lessons presented in a foreign language (English) by the CLIL method at elementary school level and to determine the difference between this educational climate and that found in mathematics lessons presented in the students' mother tongue (Czech). This research was designed to test our hypothesis, based on our previous experimental teaching, that the teaching of mathematics in a foreign language by the CLIL method is evaluated by participating students in a positive light, and to uncover significant differences regarding the students' and teachers' points of view on the learning effectiveness of each approach. It was also meant to identify possible causes of these differences, and to reveal if the implementation of the foreign language and the use of the interactive environment in mathematics lessons were viewed as meaningful and significantly improving the effectiveness of learning. The posed research questions were formulated as follows:

- Do students perceive mathematics and English separately (in the way described by Dreesmann (1982, p. 129))?
- Are students aware of the possible benefits of these subjects for their future lives?

- Do students perceive mathematics presented in English as a difficult subject full of theories and formulae, made more difficult by the environment of the foreign language? and
- Are students sufficiently motivated during such educational processes?

6. Research Methodology

The pilot research was conducted in June 2009 with 55 students from Grades 5 and 6. The aim of the pilot research was, above all, to verify if:

- students understood well the instructions regarding the work with the questionnaire and tests;
- the tests used were valid;
- students understood well the questions posed, and if only suitable questions had been chosen; and
- students were willing to co-operate in the research and to identify their attitudes towards the research.

The conducted pilot research met its purpose and verified the functionality of the research tools and comprehensibility of prepared teaching materials. Any revealed mistakes and insufficiencies were corrected or removed.

For the research purposes, each of the participating classes was divided into two separate groups. In one of the groups, mathematics lessons were presented in the traditional way as had been done in the past, and as described in the following section; while in the other part of each class, mathematics lessons were conducted in English by the CLIL method. The requirement was that the teaching of mathematics in English had to be conducted at least four times per month.

The research involved several research methods. One of these was a questionnaire designed for the investigation of the learning climate within a natural sciences subject (Grecmanová, 2008, s. 184). The questionnaire consisted of 26 statements to which respondents expressed their attitude on a 5-point scale (1-always, 2-almost always, 3-sometimes, 4-almost never, 5-never). For evaluation purposes, particular statements were divided into the following seven categories:

- K1 – teacher's enthusiasm and skill to capture students' attention;
- K2 – non-traditional way of teaching;
- K3 – support of students;
- K4 – just approach;
- K5 – meaningfulness of learning;
- K6 – adequacy of requirements; and
- K7 – clear presentation.

Other methods used in the research study were long-term observations of 10 particular lessons in each of the participating schools, video-recordings of 18 chosen lessons, and interviews with the teachers and the students. The students' attitudes were investigated with the questionnaire, and afterwards their answers were analysed using tools of descriptive statistics for testing hypotheses, in particular the non-parametric Mann-Whitney Test.

6.1 Research Environment

The research was conducted twice, first in September 2010 and then in June 2011, within six classes in Grades 6-8 (243 students in total), at three elementary schools, which closely co-operate with experts from the field of the methodology of mathematics and the CLIL methodology. Nine participating teachers were involved in the study. Groups of classes where Mathematics was presented in English by the CLIL method were denoted as group **M/A**, while the other groups, where mathematics was traditionally taught in Czech, were denoted as group **M**. The mathematics lessons covered the following topics: Natural Numbers, Decimal Numbers, Fractions, Triangles, Powers and Roots, Pythagorean Theorem, Polynomials, and Systems of Linear Equations.

In both groups, the number of girls and boys was evenly distributed. Helmke and Weinert (as cited in Grecmanová, 2008, s. 69) present in their research studies that the number of girls in a class significantly influences the classroom climate in most school subjects (the more girls in a class, the more positively the class climate is evaluated). However, Helmke also admits that there is a difference in lessons of mathematics. Girls consider the teaching of mathematics as less comprehensible and therefore they do not experience so many friendly relationships, nor hopes for success.

In all three participating schools there were no significant differences in teaching styles and working methods. The students sometimes worked on various projects, but the teachers did not intervene actively in lessons of mathematics and their organization. Also, the use of ICT was limited, as well as activating teaching methods, experiment methods, or problem-solving methods. In both of the schools, the education process could be denoted as purely transmissive. Teaching styles of the teachers could be described as authoritative, or tolerant-authoritative.

The teaching materials that were prepared within the above-mentioned project were written in English in cooperation with native speakers, and were created to be used on Interactive White Boards (IWBs). A methodological guide was prepared for each piece of material. The teaching was designed to take into account the motivation phase of the concept creation process as much as possible, and, at the same time, to take advantage of the interactive features of the IWBs and mathematical software, such as GeoGebra, to the greatest extent. According to Balacheff and Kaput (1996), an excessive use of modern technologies may negatively affect students' ability to acquire the mathematical "craft" (see also Balachef et al., 2006; Binterová & Fuchs, 2003). For this reason, the utilisation of computers and other technologies during the implementation of the CLIL method in mathematics lessons was carefully planned and the teacher/student use of technologies, and the effects of this usage, was observed with great interest by the researchers in light of Balachef's claims. The use of technology in the mathematics lessons, delivered in English, was originally meant to eliminate unnecessarily long calculations, constructions, etc., which could divert the pupils' attention from given concept explorations. As we shall see, however, the use of the technology had much greater affects on student learning, communication, and attitudes than merely providing a powerful calculation tool.

6.2 Experimental Teaching of Geometry with GeoGebra

GeoGebra, the free, open-source mathematics software primarily designed for math classrooms in secondary schools, was used by pupils from group M/A in lessons of mathematics in the English language environment. If necessary, pupils could change the language environment back to the Czech language. As mentioned in the section about ICT and the research environment, the use of computers and other interactive tools can be complicated and therefore the effort was to use it reasonably to the greatest extent without impeding pupils' acquisition of the mathematical "craft".

GeoGebra can well simulate geometric functions, as constructions benefit from dynamic illustrations of the given geometrical phenomenon. For example, one can change geometric figures while keeping the same mutual relationships between algebraic, numeric, and geometrical objects involved. This powerful feature can help teachers to demonstrate more effectively, and students to explore more creatively, several of the traditionally separated models of mathematical phenomena at the same time. Paper-and-pencil sketching or chalkboard representations could never allow for this kind of mathematical modelling. We should also mention that the use of IWBs and software can positively affect the language side of the teaching regarding both precision of the mathematical calculations and also the English language component.

The example presented in Figure 1 covers the mathematical topic *triangles.* The aim of the lesson was to introduce the concept of *height* in triangles. Making similar constructions in traditional workbooks seems less effective in developing the concept than is possible with the use of "dynamic geometry software" (DGS) that enables students to manipulate geometric shapes and figures and to illustrate the properties of the height in obtuse triangles. The aim of the activity was to discover a new piece of knowledge and to explore mutual laws connected with it. Students worked with the tool "Relation between Two Objects" enabling them to determine and to name the mutual location of lines *a* and *b* and to construct other heights in a triangle, selecting commands from the tool menu available both in the English or the Czech language environment.

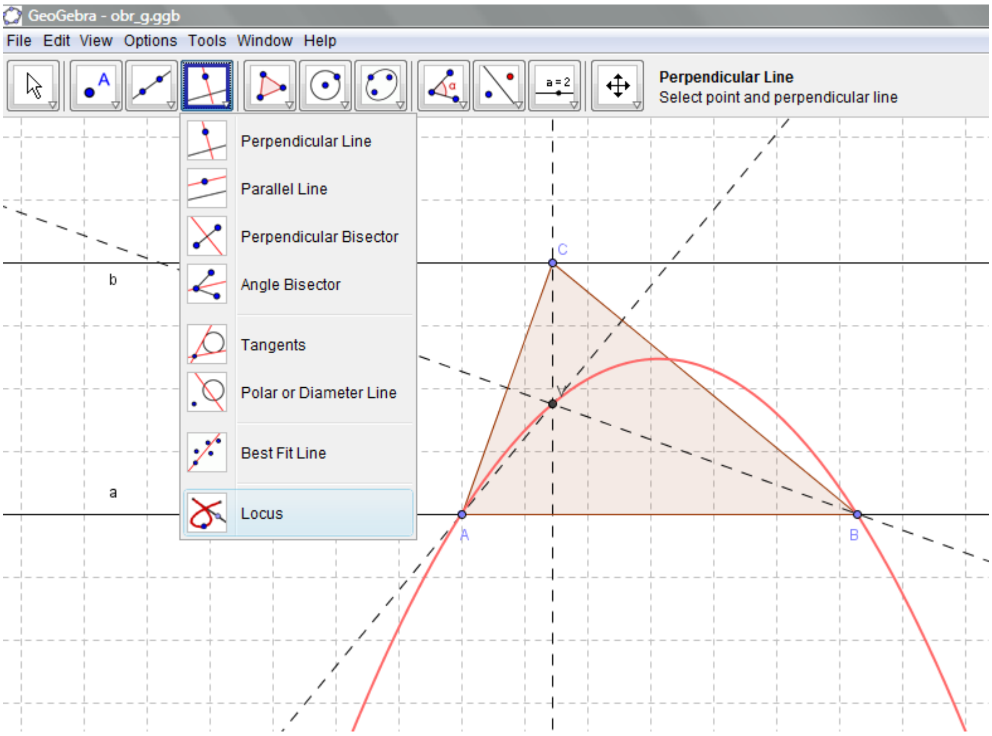

Figure 1. GeoGebra used in lessons of mathematics presented in English

Then, students were asked to construct the intersection of heights *V* and to move with vertex *C* alongside line *b*. They were asked the question, What is the shape of the curve made by the movement of intersection *V*?

This task is very difficult to solve without the help of the computer. GeoGebra enabled students to create a new dynamic image of newly discovered concepts. Using the tool *Locus* after tracing point V, students explored the newly discovered concept of the parabola.

6.3. Data Collection

The research data that was collected included marks in Mathematics and English in September 2010 and in June 2011, results of continuous tests, and data from the questionnaires. Primary evaluation used the obtained data for descriptive statistical analysis that enabled us to formulate hypotheses, which were later verified using relevant statistical software such as SPSS or Statistica. Graphical outputs were created using the program environment "R-2.13.0." Dependent variables were the English language and the use of ICT. Independent variables were pupils' knowledge, the relationship to mathematics, and the effectiveness of learning. This paper deals solely with the effectiveness of learning, based on data that was obtained from questionnaires given to 175 elementary school pupils in six classes of Grades 6-8. To make a legitimate comparison, a sample of the obtained data was limited only to those classes where there was only one teacher teaching both in Czech and in English. Group M/A consisted of 78 pupils; group M consisted of 97 pupils.

7. Research Results

The obtained data from the questionnaires were analysed in order to explore if there were significant differences in pupils' evaluation of the learning climates of groups M/A and of groups M. The results are presented in Table 1 and Figure 2—the lower values, the better evaluation of the given climate category. Both groups positively assessed clear presentation, adequacy, and enthusiasm of teachers (average values 1.84, 1.96, and 1.92), and rather negatively non-traditional ways of teaching (average value 2.77). Detailed analysis of the answers of particular categories is presented in Binterová (2012).

The null hypothesis formulated was that the answers of pupils in both groups have the same distribution, while the alternative hypothesis says that the distribution is not the same. The testing was conducted separately for each category of learning climate, at a significance of α=5 %. The obtained results are presented in Table 1.

Table 1: Evaluation of questionnaires concerning educational climate.

Educational Climate Categories	M/A	M	Deviation M/A from M	*p*-value
K1 – enthusiasm	1.92	2.46	0.54	0.000002
K2 – non-traditional way of teaching	2.77	3.09	0.32	0.003735
K3 – support of pupils	2.08	2.52	0.44	0.000211
K4 – just approach	2.06	2.65	0.59	0.000004
K5 – meaningfulness of learning	2.2	2.57	0.37	0.000671
K6 – adequacy	1.96	2.34	0.38	0.004794
K7 – clear presentation	1.84	2.12	0.28	0.003732

The calculated p-values show that in all seven categories we have to reject the null hypothesis and accept the alternative one. This means that in all seven categories the distribution is different. The mean values of answers of students from groups M/A are in all categories significantly lower (are better evaluated). Therefore, we can assume that the educational climate is perceived by these groups of students as being significantly better than was perceived by the other groups.

Observations in lessons and analyses of video-recordings revealed that pupils in the M/A groups were more motivated and more active in particular lessons than those in the M groups. Teachers in mathematics lessons presented in English with the CLIL method used more activating methods and communicated more with their pupils. The examining and testing of pupils in groups M/A showed less formalism and these pupils were directed not only to find the right solutions, but also to discuss and defend their ways of solution with their peers.

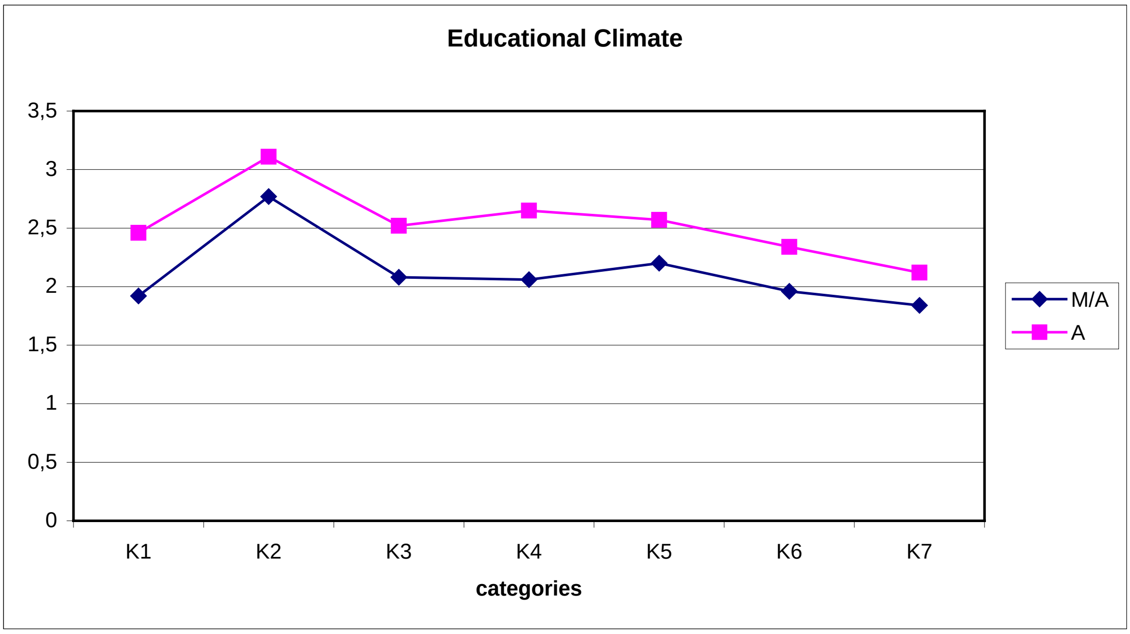

Figure 2. Average values of educational climate assessment by pupils

The observations also revealed that at the beginning of the implementation of the CLIL method, the communication in English was only limited, as there was apparent uncertainty and fear among students caused mainly by a lack of their knowledge of the new mathematical terminology which was reflected in primary obstacles in understanding the presented mathematics subject matter. However, during the experimental teaching within the different teaching environment, in comparison, the mathematics lessons presented in Czech, supported by ICT and a more cautious and careful approach adopted by their mathematic teachers, the students' concerns were significantly mitigated.

The participating students from group M/A stated in the conducted interviews that they saw the importance of the knowledge of English in their future life and that they did not perceive mathematics presented in English to be more difficult than mathematics presented in their mother tongue. Here are a few of their opinions (see also Binterová & Šulista, 2011):

- "It is good that I can practise my English. I like working with a laptop in mathematics lessons. I can learn new English words and I can explore new things with my classmates."
- "I like mathematics. Lessons are funny and not boring. Mathematics in English is good and I really like it."
- "It is one of my favourite subjects, it is funny and I can learn English well. I like very much doing geometry on a laptop."

The teachers of groups M/A used ICT support to a greater extent then did teachers of the groups M. This was caused mainly by the fact that teachers in the CLIL mathematics lessons used IWBs to also present new vocabulary and problem assignments using implemented voice recordings created by native speakers of the English language and implemented using a remote testing (voting) system which was used at the beginning or at the end of almost every lesson.

8. Conclusion

The qualitative and quantitative data analysis of this research implementation, indicates that the educational climate in lessons of mathematics presented in a foreign language (English) and supported with modern interactive technologies, is statistically significantly better than in the case of teaching the same mathematical subject matter in a mother tongue (Czech) without the use of computers and other interactive technology-based means. The learning climate in CLIL lessons was more positively perceived by students, in all its aspects and in all categories.

Analyses of video-recordings, observations, and questionnaires of the teaching process in CLIL lessons indicated more activating methods, an increase in students' activity, and more intensive communication between teachers and their pupils. Students in the M/A groups were more motivated and more active than their peers undertaking the same lessons of mathematics presented in the mother tongue and without a similar in-depth use of technology. Assessment seemed to be less formal with the M/A groups, and pupils were not directed solely to find the right solutions, but also to make comments on the solutions, and to defend their solutions and to discuss other possibilities of solving the mathematical problems.

Novotná and Hofmannová (2000, p. 228) claimed that the CLIL method in mathematics lessons is closely connected with the active engagement of pupils in the more general educational process. Their research included the use of a scale of non-verbal communicative means, various forms of representation (visualization, modelling, schematic and symbolic representations, and graphic organisers, etc.), interesting practical and attractive choices of subject matter, and suitable choices of organisational forms of teaching.

The research findings presented here support these former claims, as students undertaking lessons of mathematics in the English language, supported with interactive technologies, worked significantly harder and demonstrated a better understanding of new mathematical concepts. This way of teaching helped pupils to build their self-esteem regarding their ability to communicate and to use a foreign language practised in real situations within lessons of mathematics. For this reason, we contend that such teaching is meaningful and helps to create a positive educational climate wherein interactive ICT features of the educational process are well received by students and contribute to the overall effectiveness of the lesson.

Interviews with the involved teachers, as well as our own researcher observations, indicate that there seems to be a noticeable change in teacher beliefs and skills in terms of the teaching process. Teachers are looking for highly engaging teaching methods, and they do not fear working with new interactive environments and in this way they fight against teaching stereotypes and routine instruction. The initial concerns expressed by the involved teachers that such teaching would discourage pupils who do not like mathematics, and that it may negatively influence the climate of the lessons have turned out to be unfounded.

For more generalizable conclusions, further research of the same nature should be conducted at various educational levels, as this research was conducted only on the sample of 180 students in three elementary schools. There are still unanswered questions concerning the essence of school climate as a dynamic social phenomenon. Even though it can be concluded that this research experiment was beneficial, not only for the participating students and their teachers, but also for other teachers in the schools involved and for the school management, we are hopeful that similar approaches to mathematics and language learning will be adopted on a larger scale.

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Creating a YouTube-Like Collaborative Environment in Mathematics: Integrating Animated GeoGebra Constructions and Student-Generated Screencast Videos

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Abstract. This article discusses the integration of student-generated GeoGebra applets and Jing screencast videos to create a YouTube-like medium for sharing in mathematics. The value of combining dynamic mathematics software and screencast videos for facilitating communication and representations in a digital era is demonstrated herein. We share our experience with using these tools to facilitate mathematical collaboration, focusing specifically on the power of GeoGebra for student expression and creativity.
Keywords: dynamic mathematics software; GeoGebra; screencast; Jing; mathematics collaboration; visual mathematics

1. Introduction

A wide range of information and communication tools are available to facilitate mathematics learning in this digital era. In terms of Internet tools, students can search the web to find text- and image-based information (e.g., http://mathworld.wolfram.com) or they can view instructional videos (e.g., https://www.khanacademy.org/). They can also engage more actively by interacting with virtual manipulatives (e.g., http://nlvm.usu.edu/ en/nav/ vlibrary.html) or with tutors at math help websites through the use of text, graphics, and video media (e.g., https://homeworkhelp.ilc.org/).

The shift from Web 1.0, where users were recipients of static information, to Web 2.0, which allows users to create and interact with information on the web, is reflected in a parallel change from teacher-centred to student-centred learning environments (Delich, Kelly, & McIntosh, 2008; Downes, 2005). The focus on student-centred instruction, which emphasizes that students contribute actively to learning, is featured in publications by professional mathematics associations, new mathematics curricula, and in mathematics education research. Piccolo, Harbaugh, Carter, and Capraro (2008), for example, in exploring the nature of classroom mathematics discourse found that “students need the opportunity not only to hear what the teacher is teaching but actually converse and articulate their own understanding of the content being presented” (p. 404).

The National Council of Teachers of Mathematics (NCTM) stresses that student communication is essential in mathematics. It is “a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment” (2000, p. 60). Similar to the shift from Web 1.0 to Web 2.0 technologies, the NCTM message emphasizes that students should no longer be passive recipients of information, but instead be actively involved in contributing to learning and knowledge development in the classroom.

The NCTM’s vision of classroom discourse is reflected in a Math-Talk Learning Community framework that was first articulated by Hufferd-Ackles, Fuson, and Sherin in 2004. This framework appears as a central theme of resources for teachers (Bruce, 2007; Ontario Ministry of Education, 2008a, 2008b) in Ontario, Canada, and has guided our focus on student mathematics communication in the project reported in this paper. A key component of the framework, emphasized in Ontario Ministry of Education publications, is that “in this environment, students as well as the teacher are seen as important sources of mathematical ideas” (2008a, p. 25). In our research we focus on facilitating online digital mathematics discourse that allows students to present their ideas in meaningful ways.

Over the past two decades professional organizations for mathematics teachers (NCTM, 2000; OAME, 1993) have also championed the application of information and communication technology (ICT) to support teaching and learning. In Ontario this focus has been picked up by the Ministry of Education and the use of calculators, computers, and communications technologies is a theme running through all school mathematics curricula (OME, 2005a, 2005b, 2007). Teachers have supported this curriculum thrust and the Ontario Association of Mathematics Educators (OAME) journal, the *Ontario Mathematics Gazette,* regularly reports on classroom applications of ICT in a “Technology Corner” section (e.g., Bourassa, 2012).

Recognizing the rising educational importance of the Internet, the Ontario Ministry of Education has articulated an *e-Learning Strategy* (e-Learning Ontario, 2012). In addition to providing fully online secondary school credit courses, the strategy also encourages classroom-based teachers to adopt a blended learning approach; combining classroom instruction with student opportunities to use online resources and tools for class interaction. Although the professional and Ministry of Education calls for ICT use predated the rise of social media (e.g. Facebook, YouTube, Twitter) by a number of years, teachers are now exploring the use of these channels to support student learning [e.g., Bourassa, 2012]. Our project follows this path; providing a social media-like environment for students' mathematical communication and sharing.

Data on the "Net-Generation" suggest that secondary school students in developed countries, a particular sample set being the Canadian students involved in our project, are experienced with the Web. Across Europe and North America two-thirds of the population accesses the Internet (Internet World Stats, 2012) and of these, over 85% use social media (comScore, 2011). Internet users of ages 15–24 years are the greatest employers of these technologies; on average visiting social networking sites for more than seven hours per month (comScore, 2011). Within this domain, online video sharing, particularly through YouTube, is a major form of interaction (comScore, 2013).

The impact of social networking tools on teaching and learning is demonstrated in research that explores strategies for engaging students in learning through the use of YouTube (Duffy, 2008), considers the potential and pitfalls for using YouTube in education (Jones & Cuthrell, 2011), and questions whether or not Web 2.0 technologies actually facilitate learning (Luckin, Clark, Graber, Logan, Mee, & Oliver, 2009).

In considering what it looks like to communicate mathematically in an online or blended learning environment it is important to recognize the specialized nature of mathematics notation and representations. Hodges and Hunger (2011), for example, acknowledge that communicating with mathematical expressions is more complicated than communicating in other disciplines, but that there are tools available for facilitating communication and collaboration on the Internet. In the example we share below we demonstrate the power of free dynamic mathematics software and screencast software for facilitating mathematics communication online, and for creating a YouTube-like environment for collaboration.

2. Methods and Discussions

This project is part of a larger exploration into the possibilities for mathematics communication in a digital era. Therefore, in this section we first briefly describe how we established an online environment for mathematical collaboration and discuss student responses to this opportunity to share. Then, we focus specifically on how GeoGebra animation and Jing were introduced to students in a Season's Greetings activity. Our intention was to help students develop technical skills in a fun way prior to the holiday season. We elaborate on this approach and discuss how these skills can be carried over to tasks that more directly address mathematics concepts.

3. An Online Environment for Mathematical Collaboration

Three free tools were integrated to create an online environment for mathematical collaboration: (i) a wiki (PBworks), (ii) GeoGebra, and (iii) Jing (see Figure 1).

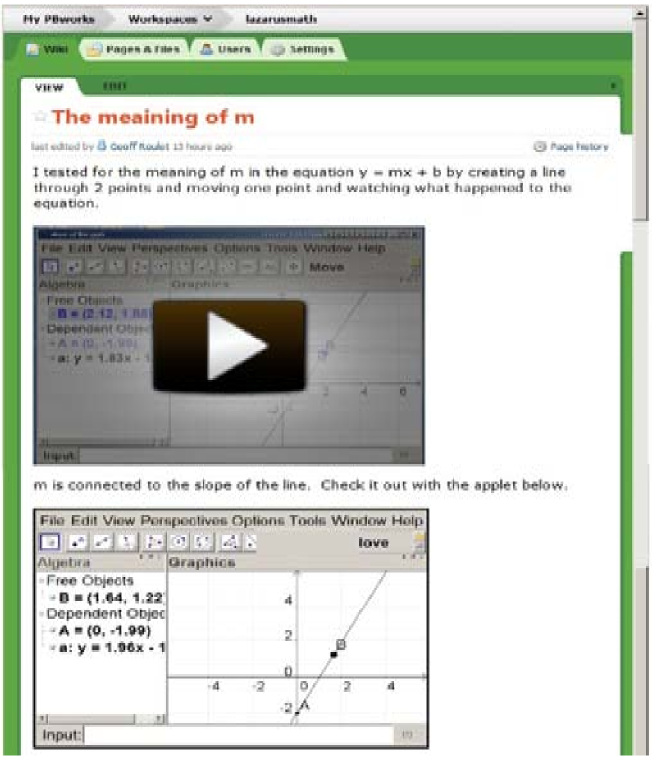

Figure 1. PBworks, GeoGebra, and Jing

Our intention was to extend the use of a class wiki, which was initially treated as a course website where students could access information (e.g., homework, deadlines), to facilitate collaboration in a math-talk learning community. In addition to contributing text to the wiki, students embedded GeoGebra applets and Jing videos which supported more specialized mathematics communication and representations.

An essential first step in this project was to establish a collaborative community in the classroom. This was particularly important since, as Lavin, Beaufait, and Tomei (2008) emphasize, "simply creating a wiki site and telling students to 'interact' (or 'collaborate,' or 'play around') on the site is unlikely to work satisfactorily" (p. 392).

We implemented this project with 23 students (11 males and 12 females) in a Grade 10 Academic (university preparation) mathematics class, taught by Lazarus in the province of Ontario, Canada. The Grade 10 Academic course "enables students to broaden their understanding of relationships and extend their problem-solving and algebraic skills through investigation, the effective use of technology, and abstract reasoning" (Ontario Ministry of Education, 2005a, p. 46). Students explore topics in quadratic relations, linear systems, analytic geometry, and trigonometry of right and acute triangles. This course provided a particularly rich context for study since our goals aligned with the course focus, and GeoGebra proved to be a particularly powerful tool for considering dynamically linked numeric (table of values), graphical, and algebraic representations of the mathematics studied.

Once students gained some experience with participating in a classroom-based math-talk community, and with using GeoGebra in class, we taught them how to embed interactive GeoGebra applets on our class wiki. Then, to support student sharing of more elaborate and creative explanations we introduced Jing screencast software. With Jing, students were able to create videos in which they recorded their screens as well as their own voices. This was particularly valuable when GeoGebra applets were animated. Before providing specific details on this latter part of the project, we present and discuss some of our experiences and the tensions that we and the students faced.

4. Net Generation or Social Media Generation?

Data concerning experience with the Internet suggests that these learners of age 15 years are experienced with communicating and sharing online (comScore, 2011), and should therefore be ready for online mathematical interaction. In the Grade 10 class, students demonstrated a wide range of technological experience, skill, and motivation. Some were quite motivated by the online aspect of the math-talk community and contributed beyond what was expected in the course. Some parents also expressed their appreciation for the wiki. They tended to like having access to course information and although the student work pages were closed to the public, a couple of parents explained that their son/daughter appreciated the wiki and was either showing them their work or expressing their interest in conversations about what they were doing. On the other hand, many students required significant technical coaching, and some were not ready to work through technical complications.

Despite arguments that teenagers have been surrounded by technology their entire lives and are therefore “digital natives” (Prensky, 2001), only a few students in this class actually appeared to be “native speakers” of the digital language of the Internet. They preferred to use the “Comment” option on the wiki for easy responses rather than engaging in the more complex task of editing pages. This is not surprising, however, since data suggest that many teenagers spend most of their time on social networking and video sites such as Facebook and YouTube (comScore, 2011) where less sophisticated short comments and the “Like” button are common responses. In a larger scale study of 2611 students in the UK, Luckin, Clark, Graber, Logan, Mee, and Oliver (2009) also found that students reported high use of social networking and file sharing sites but that their role at these sites, particularly with respect to collaborating, was relatively unsophisticated. Thus, after our experiences with student difficulties and greater reluctance than we anticipated we wondered about the question, Are we working with a Net Generation, familiar with a wide range of computer and Internet applications, or with a Social Media Generation? Introducing student-generated GeoGebra applets and Jing screencast videos allowed for student expression and creativity in an environment that seemed to more closely match their online experiences with social media sites such as YouTube.

5. GeoGebra as an Animation Tool

The animation feature in GeoGebra was introduced in a Season's Greetings activity in the days leading up to our school Christmas break. At this point, students were familiar with using GeoGebra in class and with embedding applets to the wiki. They were also a few months into the course and had various experiences with collaborating, so for this activity they worked in groups of three or four during class time, all working on one computer.

To help generate ideas we showed students a few examples, such as the animated star GeoGebra construction shown in Figure 2.

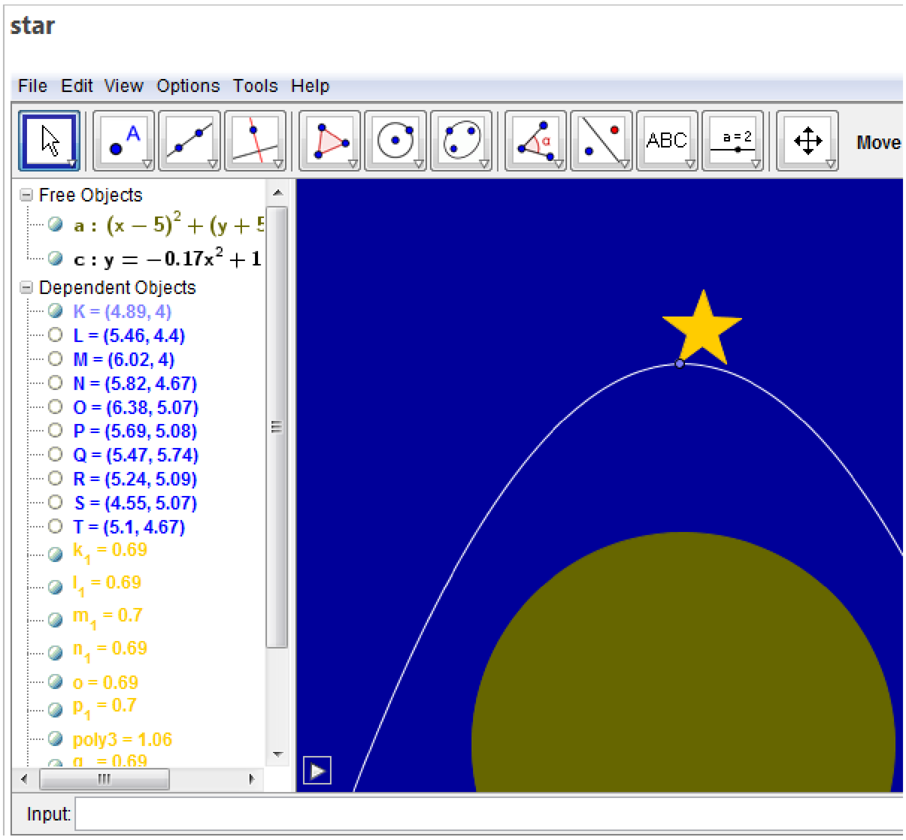

Figure 2. Season's Greetings example: An animated star

In this example the construction of the star originates at a free point on the parabola. The star can then be easily animated to follow the path of the curve by selecting the point with a right-mouse click and then selecting "Animation On" from the drop-down menu. The parabola and the point on the curve can be hidden, but showing these objects was helpful for demonstrating the animation feature in class.

GeoGebra allows users to show or hide views (algebraic, graphic, spreadsheet), objects, and toolbars, and we encouraged the groups to hide anything that did not contribute to their pictures. As the groups were finishing up their GeoGebra constructions we introduced the use of Jing. With this software the students could capture the GeoGebra window on the computer screen, recording the animation in video, and simultaneously, with a microphone connected to the computer, record a Season's Greeting message. Posting of these videos to the wiki allowed the students to share their greetings, as in our model Star video.

6. Student Expression and Learning from Animation

The Season's Greetings animation activity was a key step in introducing technical skills and in developing students' commitment to online expression and creativity. Although during initial tasks with the class wiki we found that students experienced difficulties and showed some reluctance to communicate online, they were actively engaged in expressing themselves through animated GeoGebra applets and screencast videos. They contributed a range of unique and creative constructions, of which two examples are shown in Figure 3.

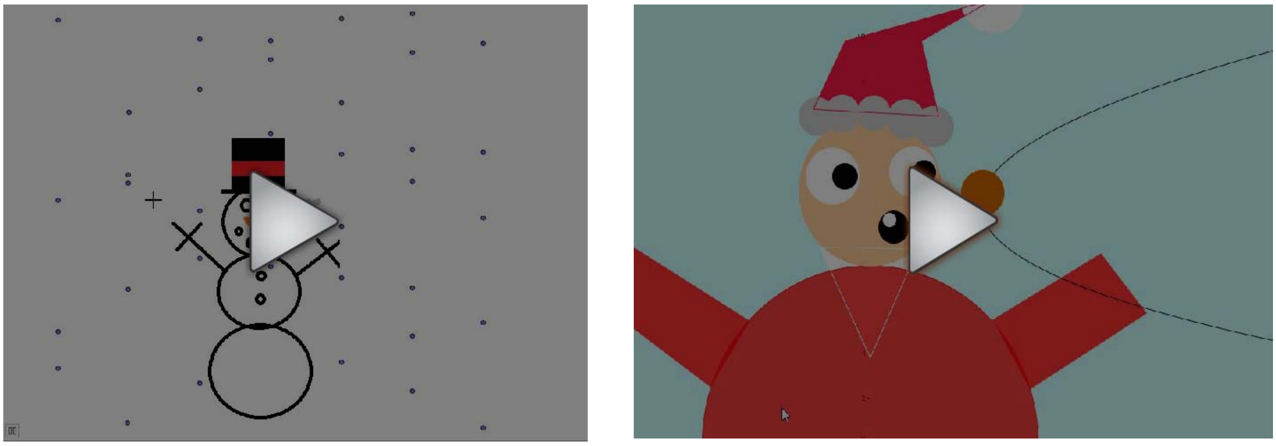

Figure 3. Animated Student Examples: "Snowman" and "Cookie!"

Various curves were employed in the students' constructions and animations. As in the images shown here, multiple circles were used, but more importantly the groups used different curves for the animated objects. In *Snowman* the falling snowflakes are points travelling along hidden vertical straight lines, while in *Cookie!* the students decided to have the cookie, the brown circle, follow an ellipse as it flew by the Santa Claus figure. This use of a variety of geometric forms provided opportunities for mathematical explorations related to a major theme of the Grade 10 course, that of linear and quadratic relations.

The opportunity for creative expression in this Season's Greetings assignment carried over into subsequent course tasks. In a final assignment for one unit, for example, students were asked to submit their solution to a problem in a form that they felt was most appropriate. Although not all students contributed Jing videos, a variety of creative responses were submitted. Some students still preferred to submit the more traditional paper-and-pencil write-up while others shared GeoGebra constructions along with text and/or Jing recordings to explain their work. One student even explored special effects in his creation of a video using a different type of video software to communicate his solution (Figure 4).

Figure 4. Promoting student expression and creativity

Beyond their school experience many students are experimenting with digital technologies. For some, opportunities to employ their ICT skills and express themselves through creative displays of work can be a motivator for mathematics study.

The potential for learning from GeoGebra animation became clearer to us after implementing the holiday greetings activity. Looking back we realize that time constraints during the period of the activity, and perhaps our focus on supporting the development of technical skills required for future course tasks, contributed to our not taking advantage of some of the rich student-generated learning opportunities.

At the time of the activity, the class was exploring non-linear second degree relations. The students knew about parabolas and quadratic equations but they had not explored other conics such as the circles in the examples shown in Figure 3 and the elliptical path of the cookie. The shapes of the curves that the groups were using and the related equations shown by GeoGebra provided avenues into exploring a wider range of second degree equations. In particular, an examination of the equations related to the shapes constructed using the geometry tools would have shown that all except for the straight lines had at least one squared term, either for *x* or *y*.

The groups' desires to produce rather elaborate animations also provided opportunities for GeoGebra skill development. In the Cookie! example in Figure 3, when the group first attempted to animate the cookie they experienced complications. Instead of the cookie moving along the ellipse, it grew very large and actually covered the entire screen—it became an "Infinite Cookie!" Class members, in creating their pictures, had discovered the "Circle with Centre through Point" tool and used this to construct multiple fixed circular shapes such as those shown in Figure 3. In the case of the circular cookie, the group continued employing this GeoGebra tool, attaching the centre to a movable point on the ellipse and having the circumference pass through a fixed point on the plane. In this case, animation of the centre point caused the radius to stretch and the cookie to grow. This unintended and, surprising to the group, outcome motivated an exploration of the other GeoGebra circle construction tools. With experimentation the groups discovered that the "Circle with Centre and Radius" tool generated the effect that they desired—a circle of fixed radius travelling along an elliptical path. Unintended animation outcomes can be strong motivators for exploration—much more motivating than direct instructions from the teacher since the "Why?" question originates with the students themselves.

7. Conclusion

While we experienced some tensions in our exploration of mathematics communication in a digital era and while we viewed the student-generated Season's Greetings activity as essentially a fun way to introduce technical skills, we did find that this task was a key step in the project. In addition to building a YouTube-like environment for collaboration in mathematics, which appears to be more closely aligned with students' online experience, introducing these skills through this activity promoted student commitment as well as a desire to show their self-expression and creativity within a mathematical environment. This discovery may have implications for future research, and for teachers who are using ICT as part of their mathematics instruction.

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