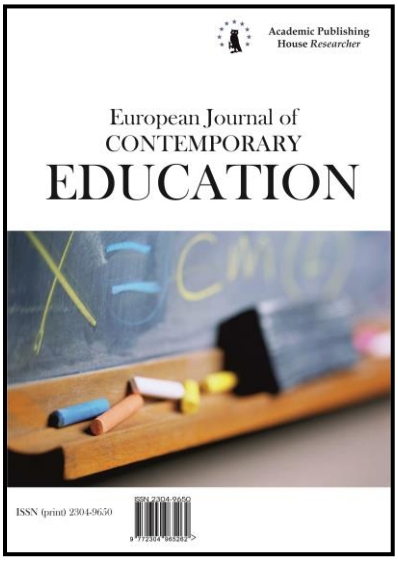
Published in the Slovak Republic
European Journal of Contemporary Education
ISSN 2304-9650
E-ISSN 2305-6746
2017, 6(4): 684-699
DOI: 10.13187/ejced.2017.4.684
www.ejournal1.com



Technology of Synergy Manifestation in the Research of Solution's Stability of Differential Equations System

Svetlana N. Dvoryatkina [a, *], Roman A. Melnikov [a], Eugeny I. Smirnov [b]

[a] Bunin Yelets State University, Yelets, Russian Federation
[b] Yaroslavl State Pedagogical University named after K.D. Ushinsky, Yaroslavl, Russian Federation

Abstract

Effectiveness of mathematical education as non-linear, composite and open system, formation and development of cognitive abilities of the trainee are wholly defined in the solution of complex tasks by means of modern achievements in science to high school practice adaptation. The possibility of complex tasks solution arises at identification of "problem zones" of mathematical education and creation of the generalized constructs of a substance of basic educational elements. As an example of a research of "problem zone" in high school mathematics the problem of visualization and a research of a phase portrait of decisions in the qualitative theory of differential equations for one or several variables functions in the continuous time is considered All stages of developed technology of synergetic effects manifestation are explicated on the example of composite educational construct development – solution stability of differential equations system. As the didactic mechanism of generalized construct substance development the authors suggest a base of professional and applied tasks and modern information and communication technologies of different level: distant environments, mathematical packages, cross-platform environments. Through mathematical and computer modeling it is possible to build the stages of adaptation and technological constructs of updating and manifestation of synergy in mathematics training on the basis of cultures dialogue. The offered technology allows to enhance educational and professional motivations of students, development quality of mathematical knowledge, contributes to efficient development of intellectual thinking operations with the possibility of self-development and manifestation of creative independence of the person.

Keywords: synergy of mathematical education, teaching mathematics, bifurcation analysis, stability of solutions.

* Corresponding author
E-mail addresses: sobdvor@yelets.lipetsk.ru (S. N. Dvoryatkina)

1. Introduction

According to conclusions of the researchers who are engaged in studying of nonlinear systems with chaotic dynamics the complexity is the integrating characteristic of system ability to the self-organization at achievement of its certain critical levels of development (N. Winer, E.N. Knyazeva, S. P. Kurdyumov, G.G. Malinetsky, G. Haken, V. E. Hitsenko, etc.) (Knyazeva, Kurdyumov, 1994; Malinetsky, 2013; Hitsenko, 2014; Haken, 2014). The research of cognitive abilities and cognitive activity of students is interfaced to learning of regularities on complex problem solutions in the process of scientific knowledge evolution (Stepin, 2008). Opening universal patterns of system self-organization of the nature and teaching process to which it is necessary to refer mathematical education, it is possible to establish the heuristic value of complex mathematical tasks in training mathematics for effective personal development as an entry into structure of the attractor in the updating context of cognitive activity. The content of mathematical education in higher education abounds in difficult, multistage abstractions of basic educational elements and procedures that create in most cases a basis for their formal development without the due organization and ensuring efficiency of cognitive processes. We see the solution of this problem in the technological maintenance of cognitive activity of students focused on manifestation of a synergy in the course of identification and updating of generalized designs of modern scientific knowledge (complex problems) with their subsequent adaptation to the content of mathematical education of future expert. It will create the conditions for bifurcation transitions to higher steps of intellectual operations of development and thinking which is trained (A.G. Asmolov, V. V. Davydov, P. Ya. Galperin, E.I. Smirnov, N. F. Talyzina, V.D. Shadrikov, etc.). Such cognitive activity of students is directly connected with research of modern achievements in science and their adaptation to a cash condition of mathematical experience on the basis of information and communication technologies and integration of knowledge from different areas. It creates the conditions for increasing of educational and professional motivation of students, mathematical knowledge and actions development qualities, effective development of intellectual operations of thinking with possibility of self-development and creative independence manifestation of the personality.

2. Methodology and technology

The synergetics which is originally created on a mathematical and natural-science basis and investigating patterns of self-organization and self-development, nature of difficult mathematical knowledge evolution includes in the consideration also questions of knowledge, education, scientific creativity, interaction with the social and cultural environment. The essence of synergetrics as integrated science consists in studying of the processes of spontaneous ordering (transition from chaos to an order) in the open systems of various nature exchanging with environment energy, information and emergence of new properties (Haken, 2004). Thus, expansion of synergetic methods to various sciences is effective when it is required to consider self-organization and self-development, integrated characteristics and difficult constructs. Application of synergy methodology to the process of education will provide a self-organization, self-development and high-quality change of person's identity through the creation of saturated information and education environment (Smirnov, 2016) in which the processes of generation of knowledge of students, their productive creativity, and awakening of own forces and ways of development are possible. According to G. Haken (Haken, 2014) upon transition from chaos to an order in all phenomena arose similar behavior of elements, which he called cooperative, synergetic effect. Identification of probabilities of manifestation of synergetic effects and mechanisms of self-organization of the personality is possible by means of the developed E.I. Smirnov of innovative technology of synergy manifestation in mathematical education, "the personality representing readiness for stage-by-stage development of mathematical activity uniting the synergetic effects expansion of theoretical (experience acquisition), procedural activity (application and transformation of experience), personal and adaptation (development of personal characteristics, intelligence) technology components of complex problems solution in the context of personal preferences realization with a high level of educational and professional motivation development" (Smirnov, 2017).

Readiness for cognitive activity of mathematical education of students on the basis of synergy manifestation should be considered as integrative unity of personal qualities, fundamental mathematical preparation and experience of teacher. This symbiosis conducts to threshold effect of self-organization and self-development of students directed on: development of multistage generality sign – symbolical systems of high level abstraction, variety of mathematical knowledge; the successful and creative solution of professional focused tasks with a support on innovations in design and self-organization of educational, training and diagnostic activity during joint comprehension of difficult problems of modern achievements in science (Smirnov, 2017; Smirnov, Burukhin, 2017).

Technological maintenance of cognitive activity of students in training of mathematics with synergetic effects is based on stage-by-stage disclosure of difficult essence of some generalized construct of scientific knowledge to research of "a problem zone" of education by means of mathematical and computer modeling in the conditions of mathematical, information, natural-science and humanitarian cultures dialogue. As a research example of "a problem zone" of high school mathematics we will consider a task *of stability theory of differential equations solutions* in the context of the adaptation to cash experience of mathematical knowledge of students.

In article (Smirnov, 2017) E.I. Smirnov revealed and characterized four stages of synergy manifestation of mathematical education on the basis of mathematical, information, natural-science and humanitarian cultures dialogue: preparatory, substantial and technological, control and correctional and generative – reformative stages. It is presented in Figure 1 the columns of stages coordination of manifestation of the essence of basic educational elements of "problem zone" and the stages of synergy manifestation of mathematical education.

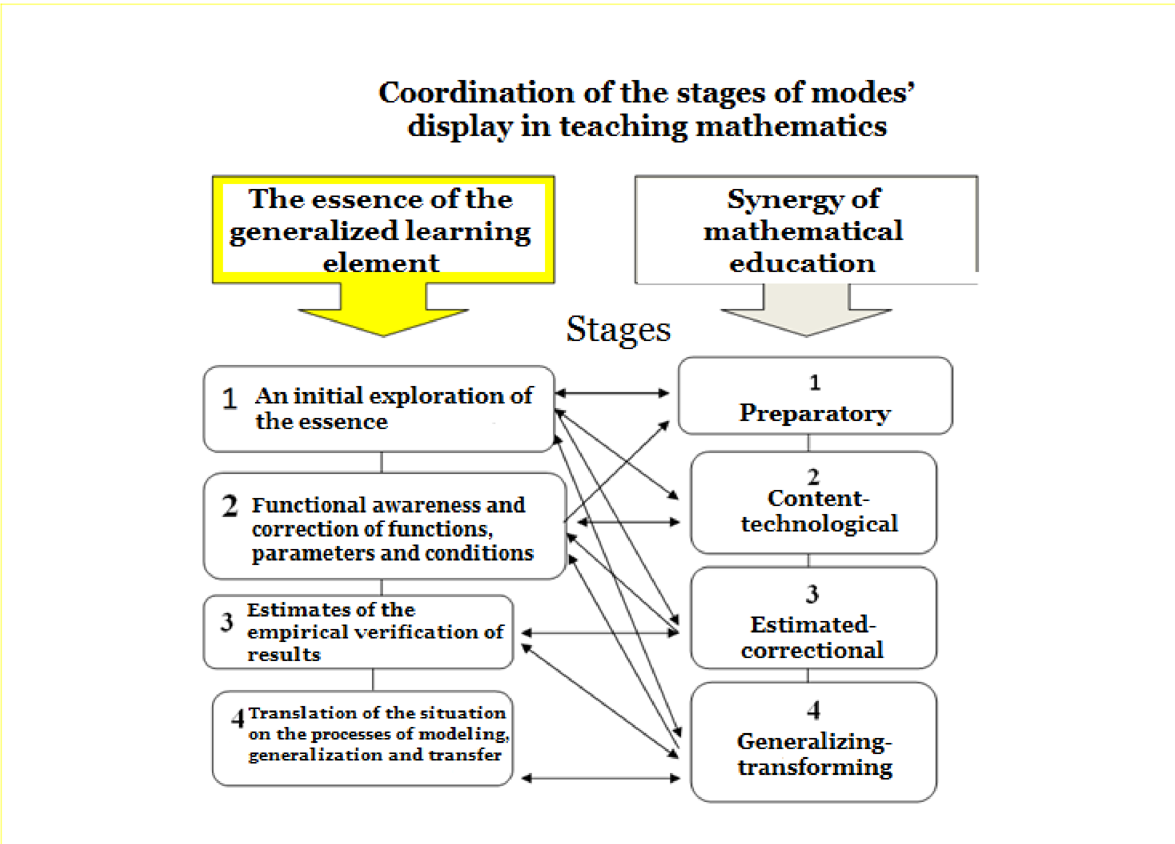


Fig. 1. Stages coordination of basic educational elements of manifestation essence of "problem zone" and mathematical education synergy

Stage I. Preparatory and organizational

Main objectives of this stage: to reveal the problem points and difficulties in successful achievement of cognitive mathematical activity of students; to actualize and create the thesaurus of mathematical education synergy: fluctuations, bifurcation points, strange attractors, etc.; to reveal the features and preferences at students in thought processes, motivation and a reflection, creativity and communicative activity; to create the steady motives of search and development new in cognitive mathematical activity.

The main result of this stage is the *definition and updating of "problem zone"* of higher mathematics (Smirnov, 2016) by means of search and research of generalized construct of scientific knowledge with the subsequent adaptation to the cash level of mathematical knowledge and methods. For example, *the problem of visualization and research of a phase portrait of decisions in stability theory of differential equations system* for functions of one or several variables by means of computer and mathematical modeling can be considered. The mathematical part of high-quality research of the equations consists in search of topological structures into which the phase portrait of system is decomposed. The applied part consists in comparison of these phase portrait structures to various processes or objects (social and economic, psychology and pedagogical, etc.) together with carrying out the bifurcation analysis (Milovanov, 2005). However the full qualitative analysis of differential equations is not always expedient. Therefore within educational process it is enough to be limited to knowledge of stability (instability area) of system and their interpretation for practice. In this regard, the essence of generalized construct of difficult knowledge consists in *characterization of solutions stability and the analysis of bifurcation transitions in topology of parameters phase portraits of an equilibrium state*. The following definition is lies in the base of stability theory: *Let vector function f and all partial derivatives $\partial f_i / \partial y_k$ be continuous on t, y. The solution $\hat{z}(\bullet)$ of normal system of differential equations $\partial y(t) / \partial t = f(t, y(t))$, where $y(t) = (y_1(t), y_2(t), ..., y_n(t))$, $f(t,y) = (f_1(t,y), ..., f_n(t,y))$ is called the stable Lyapunov, if for $\forall \varepsilon > 0$ there is such $\delta > 0$, that if $\partial y(t) / \partial t = f(t, y(t))$, $\| y(t_0) - \hat{z}(t_0) \| < \delta$, so $\| y(t) - \hat{z}(t) \| < \varepsilon$. If for $\delta > 0$ there is one solution $y_i(t)$, i=1,2,...,n, where inequalities are not fulfilled, so the solution $\hat{z}(\bullet)$ is called unstable.*

Motivational field: visual, computer and mathematical modeling of professional and applied situations in the treatment of concept "stable solution of differential equations system" (model of Lotka-Volterra competition (the competition of firms, growth of the population, number of warring armies, change of ecological situation, development of science), models of Lancaster, Lotka-Volterra with logistic amendment, Holling-Turner, Kolmogorov, Rosenzweig - MakArthur).

Forms: problem lectures, seminars, work in small groups, scientific seminars.

Means: mathematical and computer modeling with application of MS Excel, mathematical packages (MAPLE, MatLab, MathCad, Octave, SMathStudio, Scilab, GNUOctave, etc.), Web-resources.

Tasks for updating of "problem zone":

1. To investigate by means of mathematical modeling, at what values of parameter α of rest point (a special point of a steady state) of linear differential equations system

$$\begin{cases} \dfrac{dx}{dt} = -3x + \alpha y, \\ \dfrac{dy}{dt} = -\alpha x + y. \end{cases}$$

is stable or unstable?

2. To determine the character of rest point in depending on value of α.

3. Give an example of mathematical model in which there are not damped oscillations. Is it possible to consider this model suitable for the description of the processes connected with thinking, education, etc.? Check the solution of this system for stability.

4. To offer the model of two goods exchange of two producers. At what values of parameters is instability of the solution observed?

5. To offer the model of coordination of supply and demand at stable prices. At what values of parameters is there a bifurcation?

6. To offer creativity model having established possible connection between logical organized thinking and intuitive. Is it possible to take a trajectory of stable focus for creativity process which can be known or unknown? Whether bifurcation transition from a stable limit cycle to steady focus with an unstable limit cycle is possible in this case?

7. Whether it is possible to consider that in model of the competition of two types of populations (for example, dinosaurs and mammals) the death of population of dinosaurs resulted from regularization of their attractor by stable limit cycle.

Example 1. We will pass to the definition of basic designs of mathematical modeling for solution of the problem in tasks 1-2 by small groups of students.

This example represents the system of linear ordinary differential equations of first order. At first we will work out the characteristic equation for this system:

$$\begin{vmatrix} -3-\lambda & \alpha \\ -\alpha & 1-\lambda \end{vmatrix} = 0.$$

We will transform the left part in last equality by presented it in the form of polynomial which is spread out on degrees λ :

$$(-3-\lambda)\cdot(1-\lambda)+\alpha^2=0$$

and present the square trinomial standing in the left part in a standard form:

$$\lambda^2+2\lambda+\alpha^2-3=0.$$

We will enter designations for coefficients of quadratic equation:

$$a_0=1,\ a_1=2,\ a_2=\alpha^2-3$$

and will make Gurvits's determinant of these coefficients (Burkin, Melnikov, 2007):

$$\Delta=\begin{vmatrix} 2 & 0 \\ 1 & \alpha^2-3 \end{vmatrix}.$$

We will define signs of angular minors of this determinant now. Obviously that $\Delta_1=2>0$ and $\Delta_2=2(\alpha^2-3)$. In that case it is necessary to check a necessary and sufficient condition of stability. It will be carried out only under a condition $\Delta_2=2(\alpha^2-3)>0$. Applying a method of intervals to this inequality, we will receive: $\alpha\in(-\infty;-\sqrt{3})\cup(\sqrt{3};+\infty)$. We will find a discriminant and characteristic numbers of this system of differential equations, i.e. roots of the written-out earlier quadratic equation $\lambda^2+2\lambda+\alpha^2-3=0$. So we have:

$$D=4-4\cdot(\alpha^2-3)=4\cdot(4-\alpha^2).$$

1) Put first D>0, i.e. $4-\alpha^2>0$. Obviously in this case $\alpha\in(-2;2)$. Let us write the characteristic numbers in the form:

$$\lambda_{1,2}=\frac{-2\pm\sqrt{4\cdot(4-\alpha^2)}}{2}=-1\pm\sqrt{4-\alpha^2}.$$

Let us consider two parts:

1.1. If $\alpha\in(-\sqrt{3};\sqrt{3})$, so numbers $\lambda_1=-1-\sqrt{4-\alpha^2}<0$, $\lambda_2=-1+\sqrt{4-\alpha^2}>0$ will be have the different signs. Means, the rest point of system is unstable *(saddle)*. We will check the correctness of solution on the basis of computer modeling using well known software products (the program was created in Adobe Flash using Action Script programming language) (Loginov, 2005).

In Figure 2 the block scheme of algorithmization of computer model where

“ *dsolve* “ – function of the solution of Cauchy task by some numerical method (Euler, Euler-Cauchy, Runge-Kutt), “ *line*” – function of drawing of a piece on the plane.

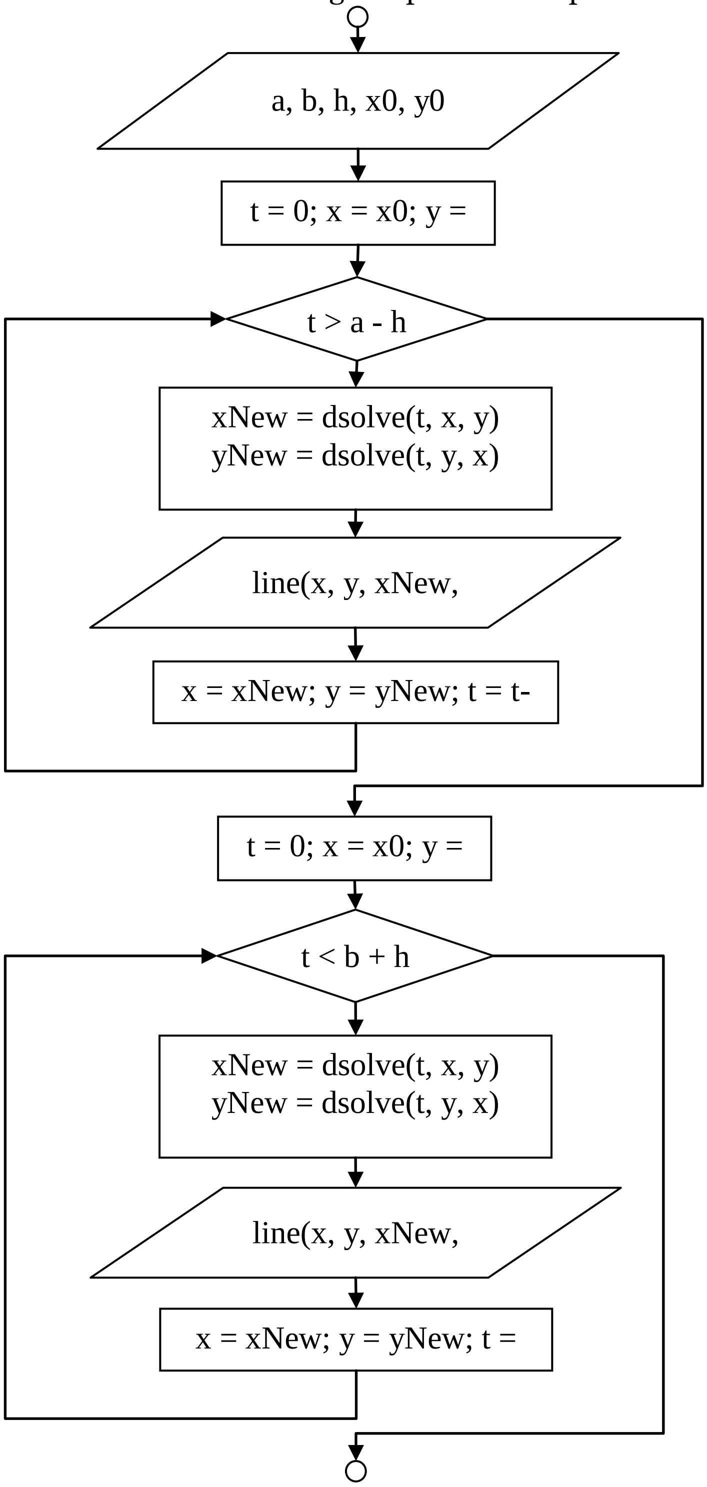


Fig. 2. Block –scheme of computer model algorithmization

Geometrical interpretation

Let us $\alpha = \sqrt{2}$, $\alpha = -\sqrt{2}$, so the systems will have the following forms:

$$\begin{cases} \dfrac{dx}{dt} = -3x + \sqrt{2}y, \\ \dfrac{dy}{dt} = -\sqrt{2}x + y. \end{cases}$$

$$\begin{cases} \dfrac{dx}{dt} = -3x - \sqrt{2}y, \\ \dfrac{dy}{dt} = \sqrt{2}x + y. \end{cases}$$

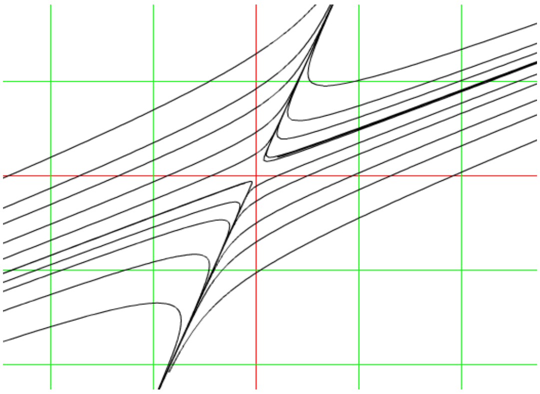


Fig. 3.1a

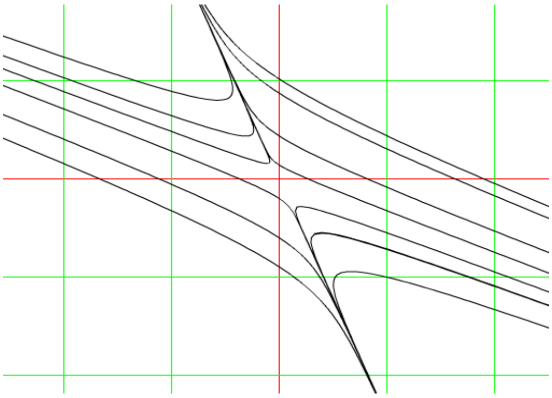


Fig. 3.1b

1.2. If $\alpha \in \left(-2;-\sqrt{3}\right) \cup \left(\sqrt{3};2\right)$, so $\lambda_{1,2} = -1 \pm \sqrt{4-\alpha^2} < 0$. In this case we have the stability of trivial solution of the system. Rest point is *stable node*.

<u>*Geometric interpretation*</u>

$\alpha = -1{,}8$

$$\begin{cases} \dfrac{dx}{dt} = -3x - 1{,}8 \cdot y, \\ \dfrac{dy}{dt} = 1{,}8 \cdot x + y. \end{cases}$$

$\alpha = 1{,}8$

$$\begin{cases} \dfrac{dx}{dt} = -3x + 1{,}8 \cdot y, \\ \dfrac{dy}{dt} = -1{,}8 \cdot x + y. \end{cases}$$

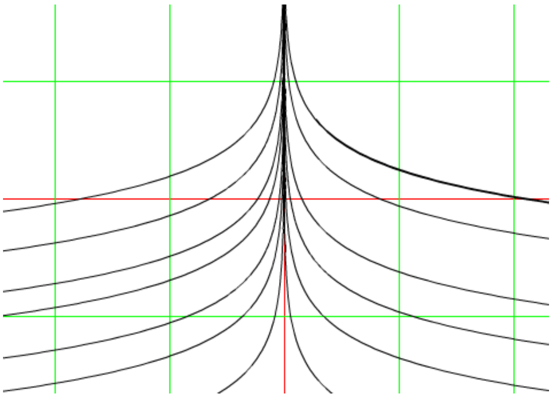


Fig. 3.2a

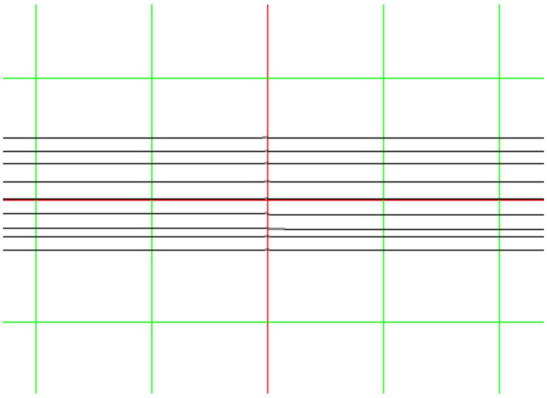


Fig. 3.2b

2) Let us consider *D*=0, i.e. $4-\alpha^2 = 0$. It is clear that $\alpha = \pm 2$. In this case $\lambda_{1,2} = -1 < 0$ and we have a stable solution. Character of rest point is *diacritical stable node.*

<u>*Geometrical interpretation*</u>

$\alpha = 2$

$$\begin{cases} \dfrac{dx}{dt} = -3x + 2 \cdot y, \\ \dfrac{dy}{dt} = -2 \cdot x + y. \end{cases}$$

$\alpha = -2$

$$\begin{cases} \dfrac{dx}{dt} = -3x - 2 \cdot y, \\ \dfrac{dy}{dt} = 2 \cdot x + y. \end{cases}$$

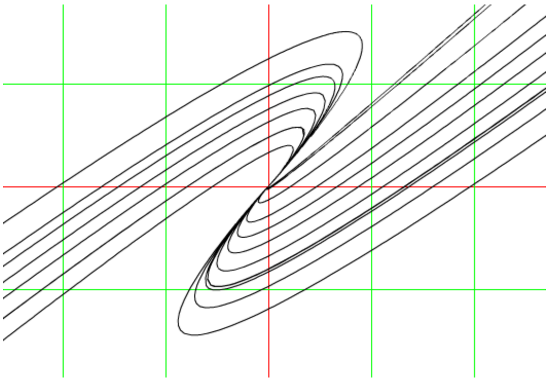


Fig. 3.3a

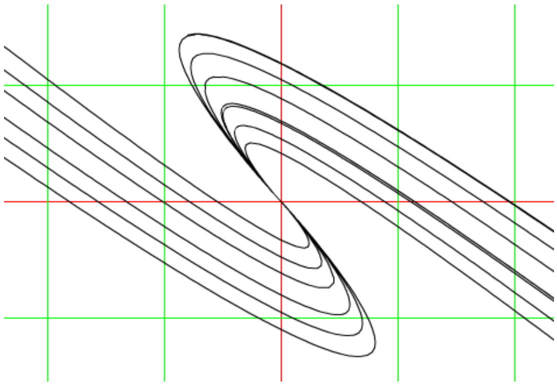


Fig. 3.3b

3) Let us consider the last case *D*<0, i.e. $4-\alpha^2<0$. Then $\alpha\in(-\infty;-2)\cup(2;+\infty)$. Obviously that both characteristic numbers $\lambda_{1,2}=-1\pm\sqrt{4-\alpha^2}$ will be a complex numbers. In this case the trivial solution is asymptotic stable and character of rest point is *stable focus*.

Geometric interpretation

$\alpha=4$

$$\begin{cases}\dfrac{dx}{dt}=-3x+4\cdot y,\\ \dfrac{dy}{dt}=-4\cdot x+y.\end{cases}$$

$\alpha=-4$

$$\begin{cases}\dfrac{dx}{dt}=-3x-4\cdot y,\\ \dfrac{dy}{dt}=4\cdot x+y.\end{cases}$$

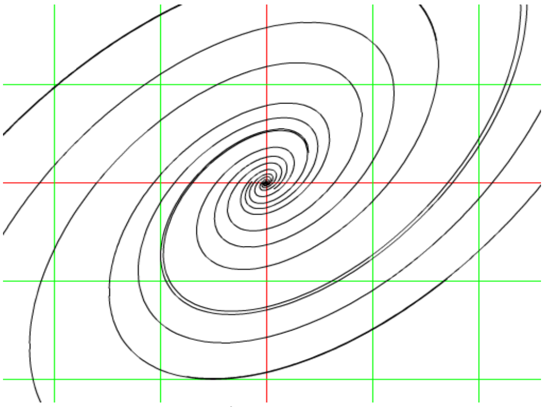


Fig. 3.4a

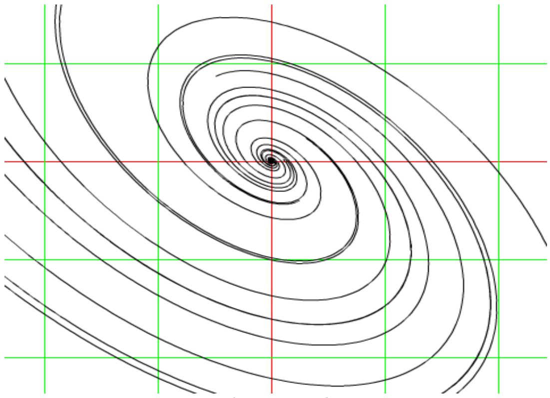


Fig. 3.4b

Remark. This case is of special interest $\alpha=\pm\sqrt{3}$. So we have incomplete square of characteristic equation $\lambda^2+2\lambda=0$. Then roots will be $\lambda_1=0$, $\lambda_2=-2$ and the solution will be stable.

For cases $\alpha=\pm\sqrt{3}$ we will have the following trajectories:

$\alpha=\sqrt{3}$ $\alpha=-\sqrt{3}$

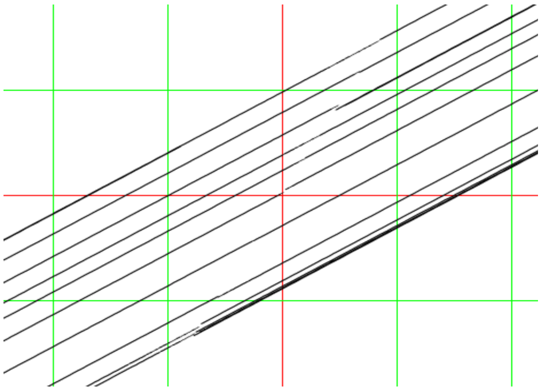


Fig. 3.5a

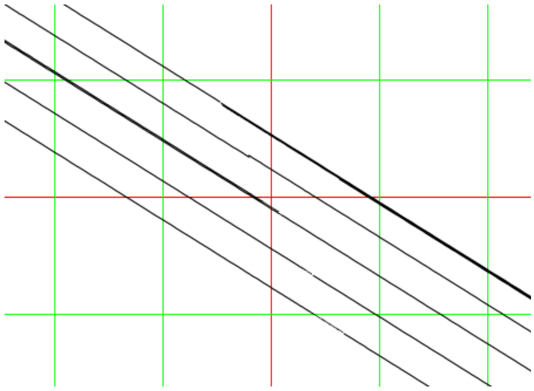


Fig. 3.5b

In the course of *updating of attributes of a synergy* at this stage the thesaurus relating to the theory of bifurcation analysis was first created. The term "bifurcation" became one of the key concepts which entered it. The etymology of this word is connected with the Latin word "bifurcus" which means *"doubled"*. We will bring one of its known definitions. Bifurcation — the concept used in some sections of mathematics in relation to situations when some object depends on parameter λ (not necessarily scalar) and in any vicinity of some value λ_0 of the last (bifurcation value, or a bifurcation point) the studied qualitative properties of object aren't identical to all λ (Prokhorov, 1988).

From the point of view of the systems of differential equations theory (and other functional equations, for example, integrated, integrated-differential) this concept describes the behavior of dynamic system consisting in acquisition of new quality of this system solutions at small change of its parameter (parameters). When parameter reaches some critical value, there are so-called *points of bifurcation* (differently *points of* possible ways *branching* of system development).

Secondly, *tasks for small groups of students* are developed:

- *for groups of analysts:* to carry out the display of equations; to investigate the solution on stability; to conduct the mathematical modeling and research of a phase portrait of system; to carry out the analysis of balance points, line isoclines and their movements in phase space;

- *for groups of programming and computer design:* to develop and realize the computer program of phase portrait visualization at various values of parameters (or for possibility of the fullest analysis of bifurcations cascade, for example, in Lotka-Volterra system, Kolmogorov model, etc.).

Stage 2. Substantial and technological —manifestations of mathematical education synergy directed on the development of adaptation stages of generalized construct of "problem zone" of higher mathematics to initial conditions of mathematical knowledge and ways of student's educational activity. Such activity of students is realized with reflection and research of technological parameters of functioning of adaptation system and receiving of new results. In this case the generalized construct of “problem zone” is "*stability of the decision of system of the differential equations*". It is actualized by means of computer and mathematical modeling of processes with strange attractors and difficult predicted.

Main objectives of this stage: to master by means of mathematical and computer modeling substantial constructs of adaptation methods of generalized scientific knowledge to initial conditions of mathematical knowledge and ways of professional activity of students; to reveal and prove new mathematical results during development and research of generalized construct; to provide the high level of professional motivation of students; to reflect the thesaurus of mathematical education synergy during research activity; to develop probabilistic style of thinking and creative independence of students on the base of integrative knowledge and procedures development, designing of the contents, stages, basic and variable characteristics of experience; to develop the abilities to adapt, be improved in social communications on the basis of mathematical, information, natural-science and humanitarian cultures dialogue.

***Motivational field*:** mathematical and computer modeling of professional and applied situations of the concept interpretation "stability of the solution of differential equations system with nonlinear members" (self-oscillation – Hopf's models, Van-der-Polya, three-specific competition, etc.; stochastic fluctuations – Lorentz, Rösler and Rikitake's models).

Forms: mini-lecture, laboratory and settlement occupations in small groups, seminars and scientific conferences, design methods, remote discussion forums.

Means: mathematical and computer modeling, mathematical packages (MAPLE, MatLab, MathCad, Octave, SMathStudio, Scilab, GNUOctave, etc.); the modern languages of programming C ++, C#, Web- resources.

Tasks for updating of "problem zone" for small groups of students:

1. Give the interpretation of three-specific competition model on the basis of Lorentz's model, where $x(t)$ – flora size, $y(t)$ – quantity herbivorous, $z(t)$ – quantity of predators.

2. Interpret model of functioning of memory on the basis of Lorentz's model, where $x(t)$ –operative memory, $y(t)$ – short-term memory, $z(t)$ –long-term memory.

***Tasks for updating of synergy attributes in small groups of students*:**

- *for groups of analysts:* to reveal by means of mathematical modeling of limit objects and their characteristics depending on changes of initial conditions; to carry out the fullest bifurcation analysis of dynamic systems which include three and more equations, for example, Lorentz's model, Van-der-Polya, etc.

- *for groups of programming and computer design: develop and realize* the computer program of phase portrait visualization of dynamic system with application of modern languages of programming; to develop and realize with application of modern languages of programming the program for research of dependence of systems behavior in phase space from a variation of initial parameters (data).

Titles of projects: "Modeling of the movement of a body under the influence of linear springs in the environment having linear friction"; "Research of differential equations systems on the existence of isolated closed trajectories"; "Modeling of periodic modes arising in electric chains"; "Research of differential equations systems on emergence of Andronov-Hopf's bifurcation".

Stage 3. Estimated and correctional

This stage is characterized:

1) Assessment of methods and procedures of results finding, variation of task conditions and data, choice of an optimum solution of the problem. For example, if additional nonlinear indignations are entered into classical Lotka-Volterra system so the system can show the various cascades of bifurcations, including the gaps of honoclinic trajectories and the birth of limit cycles.

We will analyze the possibility of emergence of "bifurcation points" at change of differential equations system considered in a task 1 (stage 1). We will bring for this purpose in analytical record of this system some nonlinear components. On a phase portrait we will trace the changes in behavior of trajectories and show the bifurcation points**.**

Example 2. We will take system from a task 1 (case $\alpha = 2$). We will bring nonlinearity in the first equation. For example, we will write down system:

$$\begin{cases} \dfrac{dx}{dt} = -3x + 2 \cdot y + y^2 - 1, \\ \dfrac{dy}{dt} = -2 \cdot x + y. \end{cases}.$$

Let us construct the phase portrait using computer modeling (Figure 4).

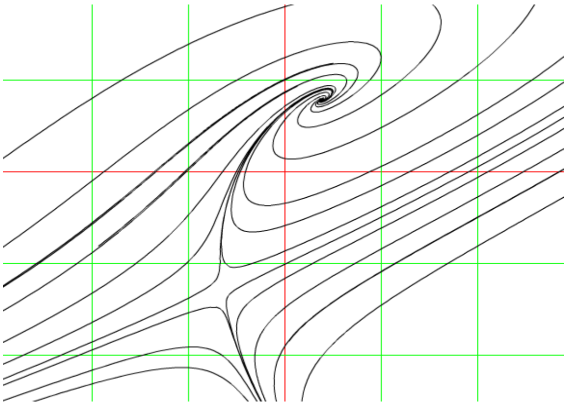


Fig. 4

We will alter the system of differential equations in which value of parameter was chosen $\alpha = 4$. We will bring nonlinear composed both in the first and in the second equations of the system. So the system of differential equations will assume the following form:

$$\begin{cases} \frac{dx}{dt} = -3x + 4 \cdot y - y^2 - 1, \\ \frac{dy}{dt} = -4 \cdot x + y + x^3 - 2. \end{cases}$$

We will have the phase portrait of this case in Figure 5.

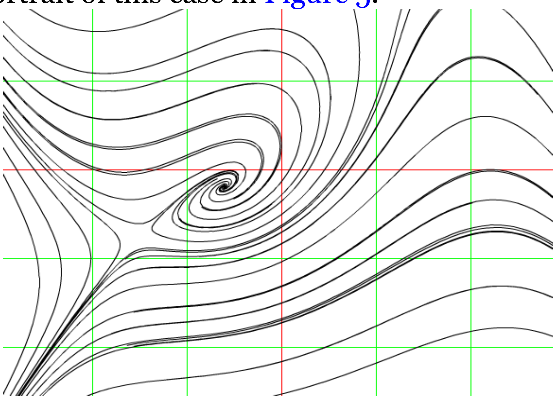


Fig. 5

So on Figure 4 and Figure 5 we show the bifurcation types "*focus-saddle*".

Let we have the differential equation system and it's phase portrait:

$$\begin{cases} \frac{dx}{dt} = -3x^2 + \sqrt{2}y, \\ \frac{dy}{dt} = -\sqrt{2}x + y^2. \end{cases}$$

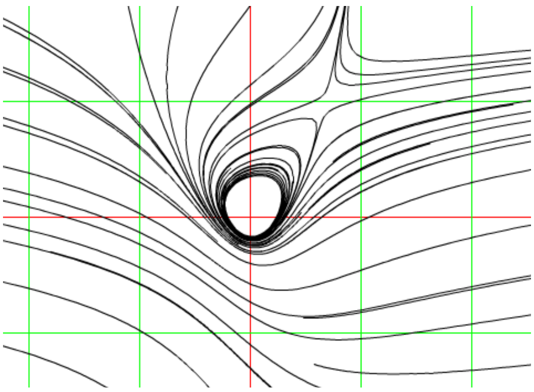


Fig. 6

Tasks: Characterize the changes which happened with differential equations system; compare its analytical record to one of earlier presented systems. Is there a bifurcation in Figure 6?

2) Monitoring of results of student's innovative activity, identification of positive and negative dynamics of parameters and indicators of cognitive activity, changes in experience and personal qualities of students.

Tasks for an assessment of cognitive activity results of students with synergetic effects:

- for groups of analysts:

The system of the differential equations and her phase portrait is given:

$$\begin{cases} \dfrac{dx}{dt} = 2x - 3xy, \\ \dfrac{dy}{dt} = 3y - 4xy. \end{cases}$$

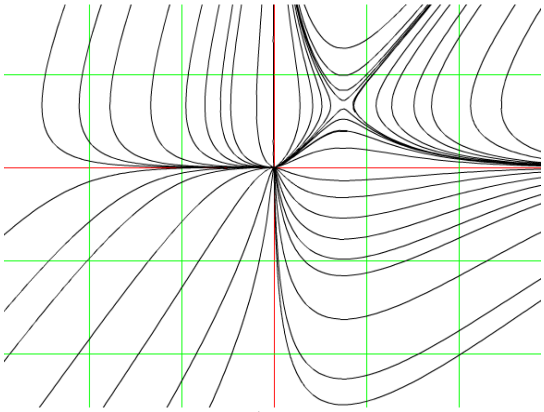


Fig.7

Specify the special points of this system of differential equations and define own values of the system; specify the stability type (or instability) by each of special points. For what description from following models serves this system of differential equations: a) Maltus's model; b) Ferkhyulst's model; c) Lotka-Volterra's model; d) Leontyev's model; e) Samuelson-Hicks's model?

- for groups of programming and computer design:

Task 1

1. Consider a situation when increase of demand for any products causes an increment of their release. Excess of demand for the first period of time t can be presented a difference

$$\underbrace{a_{11}x_1(t) + a_{12}x_2(t) + d_1(t)}_{demand} - \underbrace{x_1(t)}_{sup\,ly}.$$

Present the process of regulation for two sector economies (model "*expenses – release*") in the form of two differential equations system. As a result the system of the ordinary differential equations should be turn out:

$$\begin{cases} \dfrac{dx}{dt} = a_{11}x(t) + a_{12}y(t) + d_1(t) - x(t), \\ \dfrac{dy}{dt} = a_{21}x(t) + a_{22}y(t) + d_2(t) - y(t). \end{cases}$$

2. Present the received system in a matrix form and find its common solution at:

$$A = \begin{pmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}, \; D = \begin{pmatrix} e^{-\frac{t}{2}} \\ 4e^{-\frac{t}{2}} \end{pmatrix}.$$

Find the solution with: $x(0)=\frac{1}{6}$, $y(0)=\frac{7}{6}$.

3. Create an algorithm to solve this problem.

Task 2

1. Create the algorithm by using C++ for design of phase portrait to differential equation system with kind of nonlinearity as: $\alpha \cdot xy$, $\beta \cdot x^2$, $\gamma \cdot y^2$.

2. Build the phase portrait with the help of previous algorithm for differential equation system:

$$\begin{cases} \frac{dx}{dt} = -2x + 4xy + y^2, \\ \frac{dy}{dt} = 6y - 2xy - x^2. \end{cases}$$

3. Answer on the following questions with the help of Figure 8:

1) How many special points does this system have?

2) What are type and character of each special point?

3) Are there the bifurcation points? If "yes", then characterize the types.

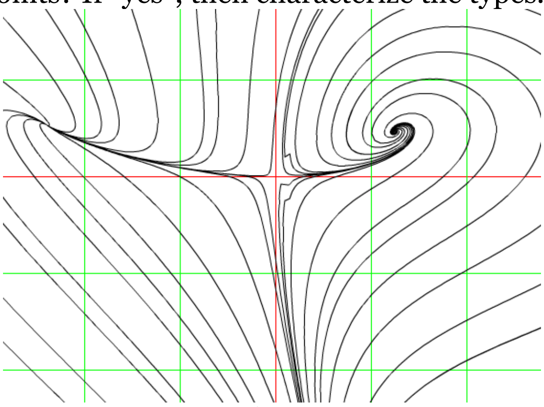


Fig.8

Stage 4. Generalized and converts

This stage is characterized by transfer of this model on various fields of knowledge where considered systems are open and there is no equilibrium. The task consists in the identification in considered systems of various fast changes, jumps, ruptures of a continuity, etc. Staying in models of systems of collective structures (focuses, limit cycles, strange attractors, etc.) by means of bifurcation analysis promotes the creative search, updating of motivation, development of cognitive activity of students in the conditions of mathematical education synergy. For example, the ability of model creation describing processes of thinking on the basis of actual information and ability to make recommendations for real practice can serve as characteristics, parameters and indicators of formation of individual style of future teacher activity. Process of thinking proceeds as follows: there are periodic fluctuations connected with steady limit cycles by receiving of brain information. Then there is a bifurcation and the system passes to steady focus with an unstable limit cycle and without cycle. During bifurcation the key information is generated and system passes to steady focuses with cycles and without. Thus it is possible to call a trajectory of steady focus as creativity (Milovanov, 2005). We will explain the ideas on a specific example.

Example 3. We will consider system:

$$\begin{cases} \dfrac{dx}{dt} = \alpha_1 x - \alpha_2 xy + \alpha_3 y; \\ \dfrac{dy}{dt} = \beta_1 y - \beta_2 xy + \beta_3 x, \end{cases},$$

where x (t) and y (t) are an information processed according to left and right by cerebral hemispheres of the person's brain; terms $-\alpha_2 xy$ and $-\beta_2 xy$ characterize the losses arising at exchange processes; components $\alpha_3 y$ and $\beta_3 x$ are the streams of information from the right hemisphere in the left hemisphere and from left in right respectively (Milovanov, 2005).

It can serve as one of possible mathematical models describing the mechanism of interaction between two cerebral hemispheres of the person's brain. This task plays an important role in such branches of modern psychological science as psycho-diagnostics and neuro-linguistic programming. Concerning the solution of this task it is important to establish the position of system balance, having described by formulas connecting coefficients of initial system and also to calculate Lyapunov's value by means of which it is possible to estimate stability or instability of model solution at the chosen values of system coefficients.

Let us put in present system: $\alpha_1 = 1$, $\alpha_2 = 3$, $\alpha_3 = 1$, $\beta_1 = 5$, $\beta_2 = 2$, $\beta_3 = 2$, then it can be right by following form:

$$\begin{cases} \dfrac{dx}{dt} = x - 3xy + y; \\ \dfrac{dy}{dt} = 5y - 2xy + 2x. \end{cases}$$

In this case the phase portrait of system will be designed on Figure 9. As we remark the equilibrium position for such coefficients choice is *unstable*.

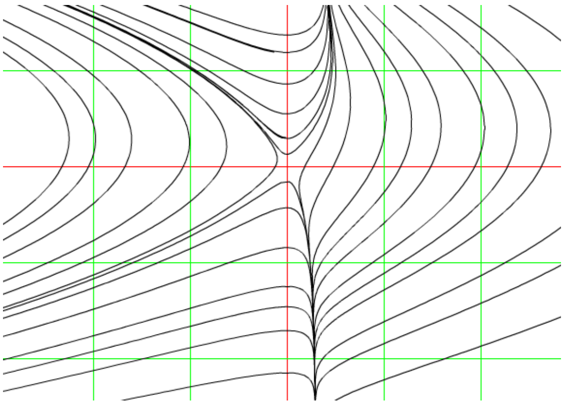


Fig. 9

3. Results of the study

The results obtained after the development and introduction in educational process of higher school confirm the thesis of efficiency the technology of synergy manifestation effects in the study of "problem zones". Phased development of complex educational construct in the adaptation process of developed system of professional–applied tasks by means of mathematical and computer modeling and design allows for enhanced educational and professional motivation of students, as well as the quality of the development and self-organization of mathematical activities. The developed technology was introduced in practice of training of the Institute of mathematics, science and technology Yelets State University named after I. A. Bunin in the period from 2016 to 2017 academic year. In the comparative analysis in the framework of the pilot study took part students of the 2-nd and 3-rd course full-time students on specialty "Applied mathematics and Informatics" at the age of 18 to 21 years. A pilot sample (n_1=12) consisted of students who studied with the introduction of innovative technologies in the discipline "Differential equations" (4, 5 semester). In the control group (n_2=11) the same profile the teaching of this discipline was carried out using traditional teaching methods (Dvoryadkina, Smirnov, 2016).

The effectiveness of technology was carried out on the basis of two criteria – the manifestation of cognitive effect (significant improvement in the quality of learning) and motivation (dominance of internal motives). Diagnostics of the quality of mastering the educational material was determined on the basis of final testing results in subject "Differential equations". The results of mathematical testing reflect the amount learned mathematical concepts, the completeness of ability formation to operate with concepts when solving professional and applied tasks, the depth of learning activity. To solve the problem of motivational effect has used a comprehensive methodology to determine the motivation of study at the University (ed. T. I. Ilyina) for diagnosing learners across the spectrum of internal and external motives: cognitive, professional, pragmatic.

For statistical analysis of experimental data used multifunction φ*– Fisher criterion allowed us to assess the significance of differences between percentages of the two samples with interesting synergistic effect (cognitive and motivational). The number of students score in the final mathematics test (K=75 and K_{max}=100) and three scales "knowledge", "mastery of the profession" and "get a diploma" the motivational criterion (M_1=9 and M_{max}=12,6; M_2=7 and M_{max}=10; M_3=7 and M_{max}=10), respectively, were considered as critical (Table 1).

Table 1. Diagnostic of cognitive and motivational effects manifestation

Groups	Cognitive criteria		Motivation criteria					
	Effect: K≥75	No effect: K<75	Effect			No effect		
			$M_1 \geq 9$	$M_2 \geq 7$	$M_3 \geq 7$	$M_1 < 9$	$M_2 < 7$	$M_3 < 7$
Control group	2(18,2%)	9(81,8%)	3(27,2%)	4(36,4%)	8(72,7%)	8(72,7%)	7(63,6%)	3(27,3%)
Experimental group	8(66,7%)	4(33,3%)	10(83%)	9(75%)	4 (33,3%)	2(17%)	3(25%)	8(66,7%)

Statistical test allowed us to reject the null hypothesis, which is that the proportion of persons who have manifested the investigated effect in experimental group more than in control group (φ^*_{exp}=2,468>φ^*_{cont}(0,01)=2,31 for indicator K; φ^*_{exp}=2,875> φ^*_{cont}(0,01)=2,31 in terms of M_1, φ^*_{exp}=1,914>φ^*_{cont}(0,05)= 1.64 in terms of M_2 and φ^*_{exp}=1,945> φ^*_{cont}(0,05)= 1.64 in terms of M_3). For internal motivation was studied the following characteristics:

– Educational -cognitive motives inherent in the learning process (interest in the profession, the desire to successfully learn, to acquire knowledge, to intellectual satisfaction, the ability to self-reliance, self-actualization, self-improvement);

– Preference of the complexity and volume of educational material;

– High cognitive flexibility in learning activities, creative solution of educational and vocational applications.

Analysis of experimental results has established the significant predominance of internal motivation (cognitive and professional motives) over external. Therefore we can conclude that the practical implementation of innovative technologies proves the efficiency of cognitive and motivational processes.

4. Conclusion

Note that ensuring the efficiency of cognitive processes in the development of mathematical activities depends on creating of saturated educational environment. This is possible in the investigation of "problem zones" of mathematical education with identification the essence of complex objects, processes and phenomena by computer and mathematical modeling means. Thus self-determination, self-actualization, self-organization and self-development reflect the complexity, openness, non-equilibrium studied meaningful constructs. In our case innovative technology is implemented to study the "stability–instability of solutions of differential equations systems". Real experience shows a significant increase in educational motivation of students and improves the quality of coherent and agreed development of mathematics and computer science in the process to adaptation of modern achievements in science.

5. Acknowledgements

The research was supported by Russian Science Foundation (project No.16–18–10304)

References

Burkin, Melnikov, 2007 – *Burkin, I. M., Mel'nikov, R. A.* (2007). An ordinary differential equation. Integration methods. Stability theory. Theory of oscillations: manual. Elec: YSU named after I. A. Bunin.

Dvoryadkina, Smirnov, 2016 – *Dvoryadkina, S. N., Smirnov, E. I.* (2016). Evaluation of the synergistic effects of knowledge integration and activities on the basis of computer modeling // Modern information technologies and it education. M.: Moscow State University.

Haken, 2014 – *Haken, G.* (2014). Information and self-organization: a macroscopic approach to complex systems / Ed. with eng. Moscow: URSS.

Haken, 2004 – *Haken, H.* (2004). Synergetics. Introduction and Advanced Topics. Springer, Berlin.

Knyazeva, Kurdyumov, 1994 – *Knyazeva, E. N., Kurdyumov, S. P.* (1994). Synergetics as a means of integrating natural science and humanitarian education /E. N. Knyazev, Sergei P. Kurdyumov. *Higher education in Russia*. No. 4: 31-36.

Khitsenko, 2014 – *Khitsenko V. E.* (2014). Self-organization: elements of the theory and social application. Moscow: URSS.

Loginov, 2005 – *Department of mathematics* at National Research Nuclear University; MIPhI (2005). Training materials: Phase portraits /Loginov A. S.URL: https://kaf30.mephi.ru/htm/fazovye_portrety_big.html

Malinetskii, 2013 – *Malinetskii, G. G.* (2013). Synergetics, interdisciplinary and post-neoclassical science of the XXI century // Preprint IPM named after M. V. Keldysh. 2013. No. 51: 36.

Milovanov, 2005 – *Milovanov, V. P.* (2005). Synergy and self-organization: the Economy. Biophysics: Monograph. M.: Komkniga, 2005.

Prokhorov, 1988 – *Mathematical encyclopedic dictionary* (1988). / Under the editorship of Yu. V. Prokhorov. M.: Soviet encyclopedia, 1988.

Smirnov, 2016 – *Smirnov, E. I.* (2016). The synergy of mathematical modeling in solving complex problems // Harmonious personal development problem in relation to specificity of modern education and socialization processes. Peer-reviewed materials digest (collective monograph) published following the results of the CXXXI International Research and Practice Conference and III stage of the Championship in Psychology and Educational sciences. London: IASHE.

Smirnov, 2016a – *Smirnov, E.I.* (2016). Methodological Foundation of a Synergy in Mathematical Education of the Teacher. *International periodic scientific journal "SWorldJournal" Pedagogy, Psychology and Sociology*. №11: 23–28.

Smirnov, 2017 – *Smirnov, E. I.* (2017). The stages of the technological support of the process of self-organization in the mathematical education of a future teacher / E. I. Smirnov, N. E. Smirnov, A. D. Uvarov. *Yaroslavl pedagogical Bulletin*. No. 3: 102-111.

Smirnov, Burukhin, 2017 – *Smirnov, E. I.* (2017). The complexity of tasks and the synergy of mathematics education / E. I. Smirnov, and S. F. Burukhin // Tasks in the teaching of mathematics, physics and Informatics: theory, experience and innovation: materials of the international scientific–practical conference dedicated to the 125th anniversary of P. A. Larichev. Vologda.

Stepin, 2008 – *Stepin, V.S.* (2008). The philosophical Foundation of synergetics // Synergetics: the Future of the world and Russia. Ed. by G. G. Malinetskii. M.: 17-23.