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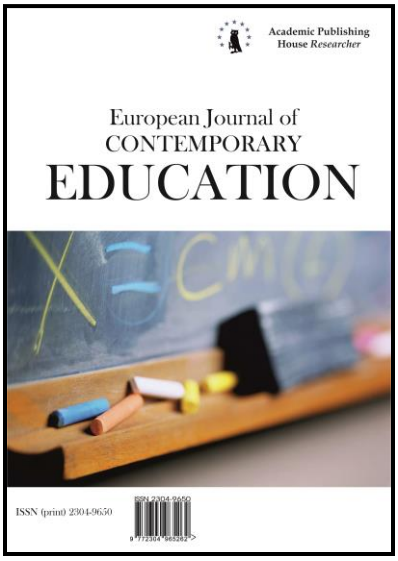

Preparing Junior School Aged Pupils for a Circle Definition: Teaching Mathematics within Physical Education Class

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Abstract

The paper presents a specific case of how interdisciplinary approach to teaching elementary mathematics can be conceived. In this context, the mathematical concept of circle is delivered through physical education environment, during a lesson aimed at speed training. The rationale for this approach is that a teacher at the primary stage in Slovakia usually teaches all subjects in a grade and can thus integrate their curricula. The basic pedagogical premises in this study are constructivism (Bruner, 1996; Dienes, 1971; Piaget, 1973) and the theory of didactical situations (Brousseau, 2002). The adopted interdisciplinary approach to elementary mathematics education is based on the model of educational reconstruction (Jelemenská et al., 2003) with a specific focus on designing a learning environment. For this purpose, the design-based research methodology was chosen as it provides for both developing relevant educational materials and conducting subsequent research. A three-phase activity was designed to guide the pupils to reflect on the concept of circle. Subsequent intervention was carried out in a physical education lesson with two classes of fourth-graders. As the design-based research methodology focuses on the causality of designed interventions (Nathan, Kim, 2009), the data were collected by the methods of participant observation and individual interview. The analysis of the obtained data indicates that proposed physical activity is potentially effective for learning the concept of circle by primary school pupils.

Keywords: elementary mathematics, interdisciplinary approach, constructivism, physical education, curriculum integration.

1. Introduction

From the theory of constructivism, it is generally known that any learner is not static in their learning. This means that since pupils endeavor to grasp new knowledge, the active factor must be present. By the active factor, we mean not only the pupils' perception but also consciousness and awareness of the pupils while they are exposed to learning material. According to the theory of

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constructivism, the pupils' active mental processes are conclusively responsible for either rebuilding existing knowledge or constructing a piece of new knowledge. To help pupils, teachers should present learning material in a way that enables the pupils attaining the knowledge easily. Kozarova and Duchovicova (2020) stated that educational material that is arranged in a non-linear way (mainly by use of graphical representations) shows improvement in pupils' results in the meaningfulness of their assertions, which is considered a substantial parameter for mathematics education.

Thought is assumed to form the basis of any knowledge. As humans use five perception modalities (vision, hearing, touch, smell, taste), the thought's nature can differ. There were multiple attempts to prove that each individual has their own modal preference, so pupils should be taught with respect to their preferred modality. This paradigm is also known as the VAK theory or the VAK model; an acronym which references to visual, auditory and kinesthetic learning style (Dunn et al., 1984). With regard to scientific facts brought forth by neuroscience, the VAK theory was recognized to be a neuromyth (Coffield et al., 2004; Pashler et al., 2008; Geake 2008; Dekker et al., 2012). Although visual, auditory and kinesthetic expressions are detected by different parts of the human brain (Dekker et al., 2012), the individual parts of the brain are mutually connected, and so the data are reciprocally transferred (Goswami, 2004; Gilmore et al., 2007). Therefore, rather than separating pupils according to their potentially preferred learning style (Sharp et al., 2008), teachers should engage all pupils with as many appropriate representations of the learning material and its various forms as possible. The visualization of math concepts is an essential element of their mental construction by pupils (Gunčaga, Žilková, 2019).

In support of this statement, Dienes (1971) argued for the perceptual variability principle, which states that learning of any concept can be maximized if the concept is displayed to pupils through many contexts and if these concepts are embodied by many forms (Gningue, 2006). The purpose of this principle is to provide pupils with multiple types of stimuli so that the acquired knowledge is not isolated and incomplete but rather deep, wide and complex.

The discipline of mathematics is supposed to be an abstract science, and so its concepts are abstract too. On the other hand, some of mathematical concepts are grounded in real-life environment. Due to this fact, mathematics can be learned by pupils who have not yet reached an appropriate cognitive level to understand abstract concepts, provided the teacher presents an abstract concept in a comprehensible way. Since the mathematics ultimate goal is to grasp abstract knowledge, Piaget (1973) proposes to follow three stages of conceptual learning: the concrete, representational and the abstract stage. To utilize this sequence requires to demonstrate the concrete model of the concept to pupils as first. After they acquired understanding at the concrete stage, the teacher can continue and explain the concept using its graphical representation such as pictures, graphs, tables etc. Once pupils understand the concept at the representational stage, the teacher may proceed to explaining its abstract characteristics.

Inspiration for the design of the proposed activity was drawn from the model of educational reconstruction for improving instructional practice (Duit et al., 2012). Primary concern of this study is to design a learning environment conducive to enhancing mathematics education. More specifically, it concerns the environment of physical education (PE) with the activities typical for PE classes.

The study provides discussion on how the concept of circle can be delivered in PE classes on the background of speed training activity. Since it is the PE content that determines the design of the intended intervention, it is necessary to specify the nature of speed training. The essential principle of speed training is to perform a physical activity with maximal intensity within a very short time and almost no resistance (Perič, 2012). Even though speed as a human ability is strongly determined by genetics, its performance level can be improved by appropriate activities. Perič (2012) proposes to include speed training into individual educational units on regular basis; the physical load should take less than 10 seconds with recess lasting at least 6 times more than the time of the physical load, 3 to 5 repetitions within 1 to 3 sets. However, these, or similar, conditions can be seldom met during PE classes as different activities should be performed to achieve given curriculum standards. Nevertheless, speed training should be included as often as possible, especially in junior school age, because this age is considered to be a sensitive period for speed development. Peráčková (2001) suggests practicing speed training right after warm-up is done, because speed development requires energy, which might be insufficiently low in later stages of the lesson. Commonly used methods for pupils' speed development are sprints and starts from various positions. Because the target group in this study is the junior school aged pupils, these exercises should be designed as competitions or games that enhance pupils' motivation to embrace speed training (Peráčková, 2001). The revised version of the National Educational Standards for Elementary Physical Education in Slovakia (2015) includes the following standards related to speed training:

- to improve the individual performance of running, tested by running 10 times between 2 stands which are 5 meters far each other;
- to apply specific means for fitness level improvement;
- to acquire the correct technique of sprinting; acceleration running and starts from various positions;
- movement games focused on speed and agility development.

For the purpose of facilitating acquisition of mathematical knowledge by pupils in an implicit way a short distance running (sprint) was utilized. Particularly, a three-phase activity was designed to lead pupils to gradually discover the concept of circle by reflecting on the distance between its center and its points. It is assumed that the activity would be conducive to discovery learning; a method with a potential to bring better educational results than the expository method (Kistian et al., 2017).

The concept of circle is a part of elementary mathematics content of fourth grade listed under the Geometry and Measurement section. However, pupils in this age often possess a prior knowledge of circle as they can encounter its models in real-life situations. The Slovak National Standards of Elementary Mathematics Education (2015) specifies pupils' performance standards regarding circle as follows:

- to discern/name a circle and area of a circle;
- to identify, mark and name the center, radius and diameter of a circle;
- to draw a circle by using a compass.

According to the TIMSS International Results in Mathematics, Slovak fourth-graders attained the lowest score in the domain of Geometric Shapes and Measures within the three distinct content domains assessed (Mullis et al., 2016). Hence, it was inferred that developing an activity which utilizes some elements of discovery learning of circle should be a reasonable step toward better mathematics education.

Before introducing the designed activity, it is necessary to clarify the concept of circle in its purely abstract nature. A circle is the set of all points in a plane that are equidistant from a given point C which is called the center of the circle. The distance r from the center to each point is called the radius.

The circle can then be graphically represented like a specific type of closed curve, as displayed in Figure 1.

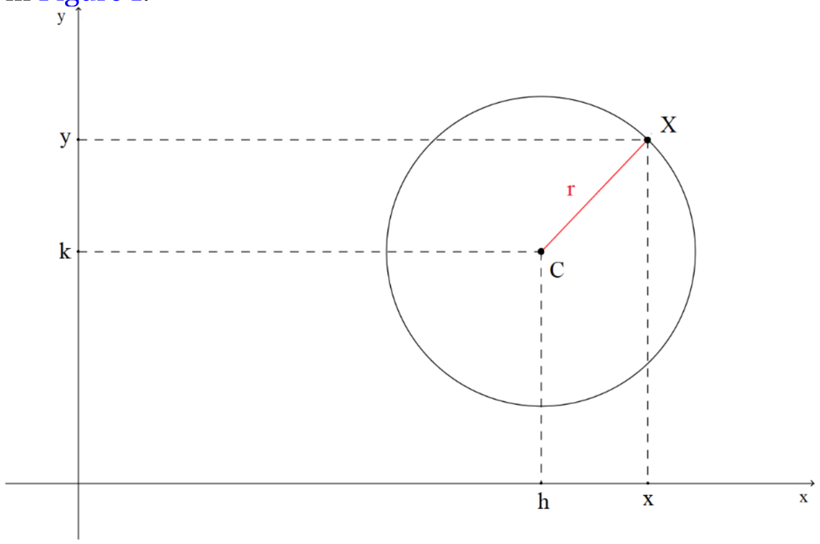

Fig. 1. A circle representation in a plane

As the coordinates of the center are $[h; k]$ and the general coordinates of all points of the circle are $[x; y]$, we can describe the set of all points of the circle with the following equation:

$$r^2 = \left|\overrightarrow{CX}\right|^2 = |X - C|^2 = |[x - h; y - k]|^2 = (\sqrt{(x-h)^2 + (y-k)^2})^2 = (x-h)^2 + (y-k)^2$$

This equation is also called General Standard Equation for a circle centered at $C = [h; k]$ with radius r.

The presented equation is the abstract definition of a circle, which can prove difficult to grasp by a junior school aged pupil and it is presented in such form much later, in the higher secondary stage of education. On the other hand, pupils commonly encounter concrete objects that remind them of a circle, such as a wheel, compact disks, Hoola hoop, traffic signs, a roundabout, a pie, a Frisbee, and many others. Based on that, even junior school aged pupils are ready to learn about this concept and possess partial understanding of it as they have some real experience with it. In a math class, they are typically exposed to a graphical representation of a circle so that they can recognize it and know how to construct it by using a drawing compass. Since they are familiar only with the graphical representation of a circle, they might tend to grasp the concept only as a type of closed curve, not engaging the actual definition. These pupils might only have procedural knowledge of a circle, which means that their knowledge is solely linked to a circle's graphical construction. To draw a circle means to draw a curved line by:

1) Taking the desired length of its radius into a compass;
2) Pressing down a needle of the compass at the point where its center will be;
3) Turning the knob on the top of the compass, so the pencil draws a circle.

The sequence of the abovementioned steps includes sensory-motor learning as well as remembering the procedure. Since its conceptual knowledge is tightly anchored to the knowledge of the definition as well, it might remain hidden out of pupils' sight. It is hence necessary to focus on how the very concept could be better demonstrated to pupils.

The pivotal part of the concept of circle is equal distance of the points from the center. Respecting Bruner's (1966) modes of thinking, the easiest way to demonstrate the distance aspect is by starting with an action-based representation of the concept succeeded by image-based observation and its reflection.

As already mentioned, the proposed intervention utilizes physical education classes, with their typical activities, for the purpose of attaining goals of mathematics education. Because the model of circle can be observed in many forms in real life, it was assumed that it could also be manifested in PE classes. The following activity was designed to guide pupils' awareness of some critical aspects of the concept of circle, particularly the aspect of distance. The intended PE class content utilizes an activity commonly used for speed training. As this activity relies on visualization of a mathematical concept, the proposed activity could have a positive impact on pupils' spatial visualization skills (Totikova et al., 2020).

2. Materials and methods

The aim of the following phases of the designed activity is to make pupils reflect on the essential property of a circle; equal distance of its points from the circle's center. The following phases introduce a didactical situation that stimulates pupils thinking leading to the new knowledge construction. According to Brousseau (2002), any didactical situation should consist of specific parts such as devolution, action, formulation, validation and institutionalization. To include all parts of a didactical situation into the activity design, it is necessary to reconcile: content of the activity, teacher's management of the class, what is communicated to the class and the way how it is communicated.

The following three-phase design combines an activity commonly used for speed training – a sprint run with the visualization of circle. The pupils are divided into several groups and stand on a start line (see groups A, B, C in Figure 2). Let us assume that each group's size is equal and individual groups do not significantly differ in performance, which can be secured by the teacher who distributes pupils into the groups. Each group is motivated to compete by the fact that they have a chance to win points for their position at the finish line.

Phase 1:

One pupil from each group starts running toward the finish line after the teacher gives a signal. Every group runs in their own path perpendicular to the finish line (shown in Figure 2).

Individual pupils compete against each other while collecting points for their groups. The teacher stands close to the finish line to judge pupils' positions at the finish line. Successively, all pupils take part in the activity. This phase is not focused primarily on teaching a circle. It is used only to bridge between regular PE activity and that which is specific for teaching a circle.

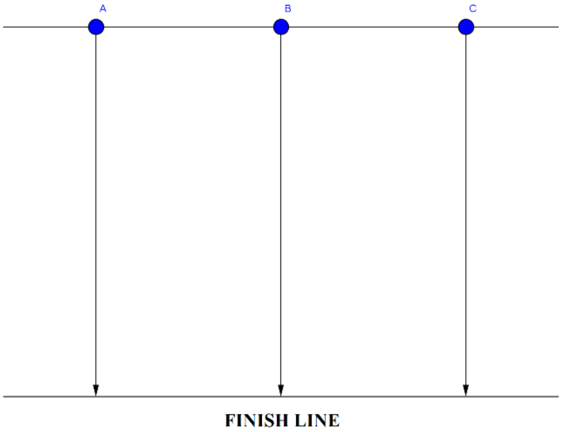

Fig. 2. Sprint run on a short distance

Phase 2:

Individual pupils run out from their starting positions. The goal is to run toward a finish line, which is the same distance as in the previous phase; however, it is narrower now (see a Figure 3). Even more, the finish line is narrower than a distance between the outer groups of pupils. This means that in their endeavor to cross the finish line, the pupils of outer groups cannot run straight, but their paths converge to the path of the group in the middle. In the real situation, the narrower finish line can be indicated by two random objects, e.g. cones, placed on the hypothetical endpoints of the line segment. Making the finish line more narrow results in:

a) probable increase of physical contact between running pupils, which can be prevented by setting an optimal length of finish line relative to the number of groups;

b) the occurrence of inequality in length of paths between individual groups.

The second result points out an important issue. The inequality is evident for the teacher who observes the situation, but it can happen that it is not apparent to pupils who participate in the activity. To conform with the principles of constructivism the pupils were left to discover this inequality by themselves. If they are unable to see it, the distance between the start line and the finish line can be shortened so as the ratio between the lengths of the path in the middle and the path on the side would become more visible to pupils. After the difference between the paths is made apparent, pupils are expected to argue that the conditions are unfair for some of the competing groups. At this point, a didactical situation is created by devolving the problem to pupils. Although pupils should be aware of the existing problem, they might not identify it accurately. Therefore, the teacher should make sure that the problem is correctly identified. After that, the phase of finding a solution takes place, and, finally, the proposed solution should be bridged with the considered mathematical concept – circle.

A solution of the arisen problem should consist of adjusting the start positions for each group, so as the distance between each start position and the finish line is equal. Modification of the length and the position of the finish line and the spans between groups A, B, C on the starting line (which can also be expressed as a length of line segments $\overline{AB}$ and $\overline{BC}$) is not allowed. To scaffold the pupils, the teacher should ask questions as: "How can we make the distance equal for each team?", "Could we make the paths of equal distance if we suppose that the finish line will not change its length and position?", "What is the distance between start positions and the finish line for each group?", etc.

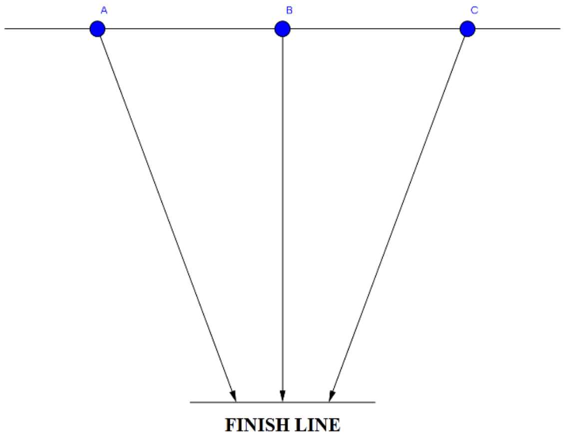

Fig. 3. Shortening of the finish line

A correct answer to the formulated problem expected from pupils is a discovery of such solution in which the start positions would be arranged, as shown in Figure 4. To guide the pupils to this answer, the teacher should steer pupils' thinking to the length factor and instruct them to compare the lengths of each group's path.

Phase 3:

Once pupils discover the expected solution to the given problem, they should prove its accuracy by executing the running activity. It is assumed that the given problem could be solved by the sequence of such actions as:

1) finding out the length of each group path;
2) moving starting positions closer to the finish line, so all groups are equidistant from the finish line.

The length of each group path can be determined according to the system of measurement adopted by the pupils. Let suppose that the pupil would decide to use the finish line's midpoint as the endpoint of their path. Subsequently, the actual lengths of paths the groups will run in are not equal since pupils from different groups do not run toward the same point (the center of the circle). The fact that they do not run toward the same point links to a safety factor, which must always be thoroughly implemented. For instance, the length of the path of the group A is shorter than the length of the path of the group B, because the line segment between point A and point where their path intersects the finish line is shorter than the line segment between point A and the center of the circle (radius of the given circle) which was used to determine new starting positions for the groups.

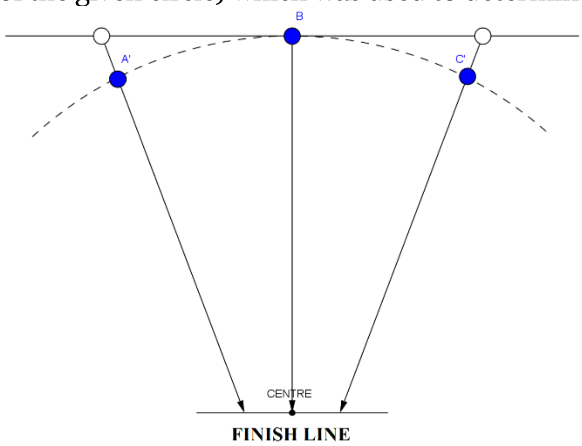

Fig. 4. New starting positions based on equal distances

The pupils carried out two activities – running from different starting positions to the finish line and measuring the path's distances, which are to be equal. The aim of the activity was to stimulate pupils' perception of a circle form in terms of its definition, i.e. as a set of all points in a plane with equal distance from the given point (center). For better understanding of this circle's characteristic, it is advised to create more starting positions and repeat the two activities: running and subsequent distance measurement (see the Figure 5).

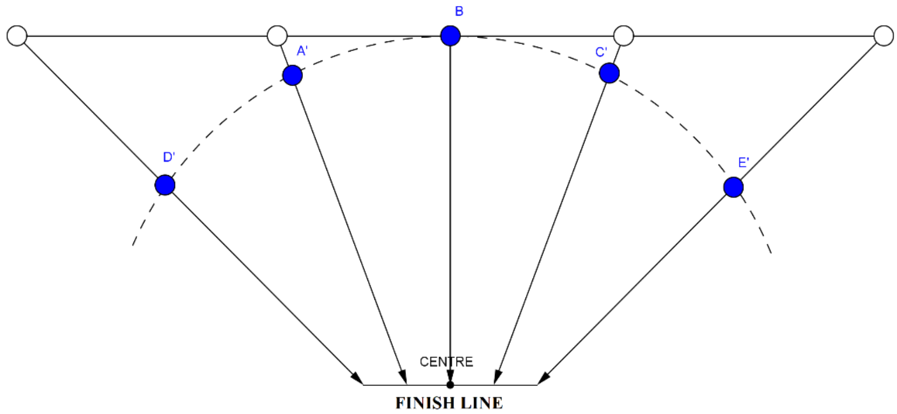

Fig. 5. Using more groups to make the arc more visible

Special attention must be paid toward safety. Therefore, the number of groups must correspond with the finish line's length to avoid physical collision of running pupils. On the other hand, creating many groups simultaneously with adjusting the finish line (to maintain the safe environment), may interfere with the aim of the activity. If every starting position is equivalent to a circle's points, centered at the midpoint of the finish line, the paths of the groups on the sides are shorter than the paths of the groups placed closer to the middle.

For instance, in Figure 5 we see that groups D' and E' are closer to the intersection of their paths with the finish line than groups A' and C', which are also closer to their endpoints on the finish line than group B. Despite the fact that the paths are not of the same length, if the finish line is not too long, the differences would not be significant enough to be observed by pupils.

Before presenting our findings, we highlight characteristics of the intervention.

The presented activity within PE classes was conducted with the fourth-graders at a local primary school. The particular local school was chosen because of the accessibility for us and because the school does not provide classes for pupils with special needs. This choice refers to the selection of a typical case, helping us to inquire within the group of pupils while they are engaged in the activity (Seawright, Gerring, 2014).

Fourth-graders were selected because they represent the oldest and potentially the most cognitively developed group within primary school population of pupils in Slovakia. Therefore, if the activity were proven educationally effective for fourth-graders, we could surmise about its effectiveness for even younger primary school pupils.

As indicated earlier, to allow pupils to see elements of circle in the form of enactive representation within the submitted activity, the number of pupils involved must be at least 4 or 5. Since the purpose of the study was to determine the impact and evaluate the effectiveness of the designed activity (Nathan, Kim, 2009), which was considered a specific case of the interdisciplinary approach, the sample size should correspond with the standard size PE class. Therefore the empirical inquiry was undertaken with a class of 20 pupils. To prove the activity reliable, we iterate the activity with another class of 20 pupils. Both classes were mixed boys and girls with the even distribution of girls and boys.

Since the current governmental regulations did not allow the activity to be carried out inside the gym due to the Covid-19 pandemic, we conducted both classes at the outside playground. The activity was scheduled at the end of each PE class. The data were collected during the classes and immediately after the class ended. To collect the data, we used non-structured participant observation, voice recording and semi-structured interviews. Voice recording was obtained during the class to record pupils' reactions and class discussion. Semi-structured interviews were conducted in order to discover how the pupils perceived the activity and the spacing of their segments, and particularly if the activity reminded them of a circle or any similar math concept.

During the activity, we avoided using words like “circle”, “center” or any other math term so that the pupils’ outcomes would be authentic.

2. Results

In this section, we present situations which occurred during the intervention and the results obtained from interviewing the pupils. The three-phase activity was undertaken gradually, as described in the previous section. Since our plan consisted of changing the finish line’s length (phase 2), we decided to use two tennis balls that we placed on the endpoints of the line segment. Subsequently, we moved them toward the midpoint of the line segment to shorten its length. Initially, we observed one practical problem that some pupils did not run across the line segment but next to it (especially pupils from the groups on the sides). This problem relates to the safety factor. In this case, the groups’ paths were converging to each other, which might have been risky for running pupils. Therefore, we had to make sure that the finish line would be long enough to fit runners from all groups running shoulder to shoulder. Because of the limited time, we decided to create 5 groups of pupils. Optimally, we would recommend 3 to 4 groups of pupils to take all safety measures to avoid physical collisions at the finish line.

After that, we shortened the finish line and let the pupils compete. Even though individual path lengths were different, none of the groups from either of the two classes argued against the new condition, except one pupil who said: “our group has the best position”. It can be assumed that the initial distance of 9 meters between the start line and the finish line (in the phase 1 and 2) was not distinct enough for the pupils to notice the differences in the length of individual paths, and as a matter of fact, none of them argued about inequity. Therefore, we proceeded to the step two, which was to move the finish line closer to the start line, so the distance was changed to 6 meters. The pupils’ subsequent performance can be summarized in these two points:

a) the gaps between the groups on the start line were shortening as the pupils wanted to face the finish line and did not run in a slant path;

b) as all groups were told to stay on their initial start positions, pupils from the groups on the sides started to have objections against their paths’ length and slope.

Since these objections against the activity were raised, the pupils were prompted to present their opinion on what they consider unfair and what should be done to make the conditions equal for each group.

However, some unexpected reactions also occurred; the pupils were concerned with the slope of the paths. Because the paths of the groups, especially the paths on the sides, were changed due to shortening the finish line, the pupils also had to change the direction of their run immediately after they sprinted out from their new starting positions. Based on that, pupils from the groups on the sides argued that it is more difficult for them to run. Even though their argument might seem reasonable, their paths were of the same distance as the other paths and their paths were not curved. However, there is one simple thing that could be done to put their minds at ease: a slight rotation of the group, with pupils staying in the line so that they would face the spot at which their path crosses the finish line.

Except for the objection related to the paths’ slope, the pupils were neither satisfied with the paths’ lengths, which were not equidistant for each group. We proposed pupils to discuss the issue and present their ideas of how the activity could become fairer for each group. In the discussion, they brought some ideas such as:

- relocate a group consisting of fast runners to a starting position on a side and move slow runners to a starting point in the middle;
- all groups should move closer to each other at the start line so they would be facing the finish line;
- the finish line should be longer so each group could run straight towards it.

After they submitted their ideas, we explained that we want to keep the finish line as it was and that spans between the groups are given and cannot be changed. Also, each group’s speed performance relates to pupils’ distribution, which was done by using the “rock-paper-scissors” game, so the participation of any pupil in the group was random.

Right after we objectively argued against these potential solutions, we tried to scaffold them toward the issue of different lengths. The key questions which incited the process of finding new starting positions for individual groups, were: “Do the distances differ?”, “How could we show it?”

After introducing these questions, the pupils concluded that they could prove the inequality by making steps and counting them or by using a tape measure. Since no one possessed a tape measure, human steps seemed to be an ideal tool. As we opted for measuring by making steps, the pupils immediately realized that each child has a different length of step. Then, we suggested that to measure precisely, only one pupil should be taking steps and counting the length of each path, which seemed like an acceptable idea for all pupils. A chosen pupil measured individual distances, which were counted as lengths of a line segment between the midpoint of the finish line and the individual starting positions. The midpoint was considered to become a center of a circle (see Figure 5), whose points are determined by fair starting positions for each group. The measurement revealed the inequality between different paths; the longest path was 12 steps long, and the shortest path (the one in the middle) was 9 steps long. Then, the pupils decided to adjust the length of each path so that each path would be equal to the length of the path in the middle. Practically, they found each group’s new starting position by subtracting its initial length by a number 9, which resulted in the number of steps they should take towards the midpoint to reach an optimal starting position. The described process of finding (locating) new starting positions can be considered peer-learning since the smarter pupils were helping other pupils to understand the process. As they succeeded in searching for the fair starting positions, we let the pupils rerun the activity to allow them to see that the new conditions were equal for each group.

Since the nature of design-based research is open and interventionist, semi-structured individual interview was preferred over questionnaire (Bakker, Eerde, 2015). Generally, interview promises to bring more subtle research data, which corresponds with in-depth qualitative research’s nature. During interviews, randomly chosen pupils were given the following task to solve:

Construct points K, L, M, N, O, P, Q, R, 4 centimeters far from a given point C.

Each interviewee was given the task sheet with preprinted point C. All interviews were conducted in a quiet room in the school with the presence of only individual interviewee and us. No time limit was imposed on the pupils solving the task.

Although no tool was given to the pupils beforehand, they immediately realized that they might need a ruler to find the missing points’ location. The ruler was given to the pupils after they asked for it, by which they manifested the necessity of using one to solve the given task. None of the pupils asked for a compass, as they have not learned yet how to use one. In most cases, they did not struggle with the task; however, some of them did not measure the length precisely due to their poor measuring skills. The idea behind this task was to determine if pupils realize a similarity between the constructed points and the starting positions in the phase 3. If they conceptually connected the constructed points with a circle – which they had not been informed of – and with starting positions in the phase 3, it would indicate that the placement of starting positions reminded them of either an arc, a semicircle, or a circle. Thus, the observed relation between the constructed points, a circle and the starting positions in the phase 3 should prove useful for teaching the concept of circle and its properties. To determine whether they associated the constructed picture with the activity, we directly asked them to look for similarities. Even more, we wanted to know whether the pupils could recognize any similarities between the activity and a circle and its properties during the intervention.

We individually interviewed 10 out of 40 pupils who participated in the designed activity.[*]

A summary of the data obtained from the interview is displayed in Table 1. The Table 1 presents different outcomes of how pupils mapped particular segments in the phase 3 and whether they indicated the connection with the concept of circle. The interviewed pupils labelled as P11-P15 and P21-P25 were from different classes.

[*] As we wanted our interview to be conducted immediately after the intervention, the number of interviewees was determined by the time constraint between the end the intervention and the end of the school day.

Table 1. Interview results and the analysis of the answers

Pupil	What does the picture evoke?	Is there any observed similarity with the last phase?	Explanation
P11	a semicircle	yes	nonspecific
P12	a circle	yes	point C reminds of the finish line; constructed points remind of groups of pupils
P13	a circle	yes	point C reminds of the finish line; constructed points remind of groups of pupils
P14	nothing	yes	point C reminds of the finish line; constructed points remind of groups of pupils
P15	a circle	not certain	none
P21	a circle	yes	the constructed points are equidistant from the point C as well as groups were equidistant from the finish line
P22	a circle	yes	the pupil related it to a different activity we were doing in the class as well
P23	a circle	yes	the starting positions remind of an arc
P24	something from the Slovak language, a snowflake, the sun	yes	the pupil related it to different activity we were doing in the class as well
P25	a circle	yes	the starting positions remind of a semicircle

When we taught the class which the pupils P21-25 belonged to, we included some more activities not mentioned in this paper. These activities were also aimed at enhancing the teaching of the concept of circle. Therefore, we assume one of these activities affected pupils P22 and P24 as their responses were related to the activity other than the one discussed here. Dismissing their responses, we see that most of the pupils (6 out of 8), especially those from the group P11-P15, quite precisely connected the activity' segments with some elements of circle. Furthermore, the pupils claimed that they had noticed the similarities with a circle even during the activity.

The research design aimed to show whether the pupils become aware of the specific associations between the activity and the concept of circle or do not associate the activity with any element of circle. Therefore, the obtained data can be sorted into two distinct categories: pupils who "associated" and who "did not associate". We had initially expected that another two subcategories would occur: a subcategory in which pupils associated only the center of the circle with the finish line and a subcategory in which pupils associated only points of the circle with the starting positions. Neither of that happened, though. This finding implies the conjecture that since the activity employs enactive representations of both the center and the points of the circle, while these two are reciprocally dependent, the pupils will always mentally associate them both at once. With this arises the question whether the starting positions of individual pupils D', A', B, C', E' (referring to Figure 5) could have influenced how these individual pupils comprehend the enactive representation of the circle. Thus, the submitted research offers ideas for the following more detailed inquiry.

According to Fusch and Ness (2015), research reaches data saturation when it can be followed up by new research. Therefore we argue for sufficient data saturation in our research. Moreover, sufficient data saturation implies that the sample size used in our research was adequate for proving the educational potential of the proposed activity (Glaser, Strauss, 1967).

Altogether, despite the abovementioned limitation of the research, the answers obtained from pupils indicate that the submitted three-phase activity can be conducive and instrumental in teaching the concept of circle.

3. Discussion

As shown above, the presented activity is potentially instrumental to a teacher when teaching the concept of circle. Naturally, this activity is based on locomotion, which usually takes place in physical education classes. In Slovakia, the teachers in the primary stage usually teach all subjects in a grade. Since the teacher is familiar with different subjects and their contents, it is assumed that such teacher is capable of integrating different subjects and their curricula. The proposed activity can be included in PE class before presenting a circle to pupils in mathematics class in the fourth grade. Contemplating the practical utilization of the designed three-phase activity, three potential issues were detected:

1) How does the design fit into the PE curriculum?

This question has already been partially addressed in the introduction section. Speed and agility development is determined by accurately structured activities which are performed frequently. This means that to accomplish corresponding curriculum standards, suggested or similar activities should be included in PE classes. On the other hand, performing the proposed activity might absorb significant time from the PE class. The class would then be purely focused on agility, and speed development and almost no other educational goals could be considered within this class. Therefore, we see some limitation in our design in the matter of accomplishing various educational goals. However, if the intended educational goal is to develop speed or agility, utilization of the proposed activity does not prevent from attaining such goals.

2) Is the suggested activity safe enough to be part of PE education?

One of the most important PE class principles is to maintain that the environment and activities are safe for pupils. Therefore, a teacher is obliged to ensure that such activity does not increase the hazard of any injury. There might be a risk of pupils' collision when they run since their paths converge to each other as they approach to the finish line (phase 2 and 3). It is therefore necessary to adjust the finish line to be wide enough for all runners to run through while each of the runners has sufficient lateral space. The National Center for Health Statistics (1973) published that at 11 years of age, the girls' chest girth ranges from 60,4 to 83,4 cm, and for boys it is from 63,3 to 83,1 cm. According to these data, we suppose that each pupil should be assigned to at least 80 cm of the finish line's width, which should theoretically guarantee that pupils should not bump into each other while running. We have employed a volleyball court in our design as it is a commonly available and well-marked area of school gyms. Since the activity is designed for a volleyball court, the maximum width of the starting line can be 9 meters, as the court is 9 meters wide and 18 meters long. To keep pupils safe, we propose the width of the finish line is relative to the number of teams. As the increasing number of the teams would make the differences between the paths' lengths less visible to the pupils, we suggest creating not more than 5 competing teams.

3) Does the activity and its phases genuinely model and represent a circle?

According to the obtained data, we suppose that the activity is designed to bridge the concept of circle when discussing the segments with pupils. This connection is based on the actual actions carried out by the pupils while they are looking for ideal starting positions.

In our research, each path's length was calculated so that each group counted steps between its initial starting position (points D, A, B, C, E in Figure 6) and the finish line's midpoint (see intermittent straight lines in Figure 6).

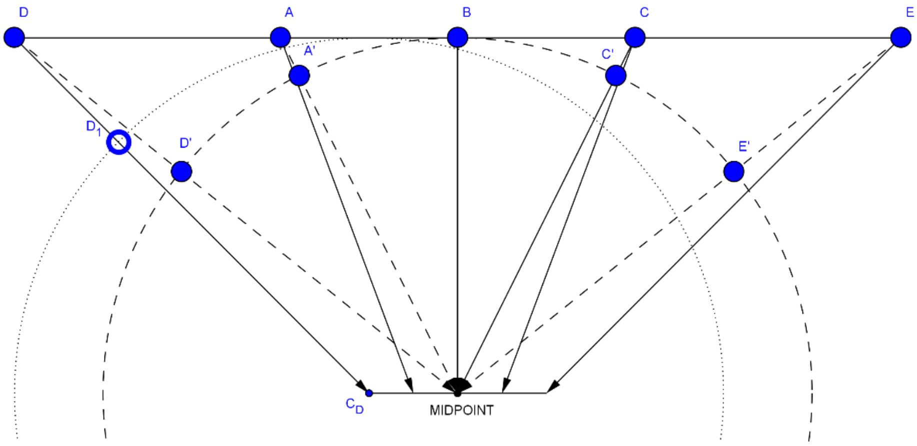

Fig. 6. A problem of different lengths

However, the groups' actual running paths are not equivalent to the paths used for calculating their lengths. As can be seen in Figure 6, according to the real measurement, group D should move their starting place from D to D'. However, pupils from the group D will not run toward the finish line's midpoint but toward a spot of the finish line, which is closest to them (point C_D in Figure 6). Since their initial running path is directed toward point C_D, a new starting position should be placed on the intersection of their initial path and the circle centered at C_D. The radius of this circle is equivalent to the line segment between the midpoint and group B's initial starting position (see point D_1 in Figure 6). We have used group D as an example, but the new starting positions of all groups except group B should be corrected if all real paths are to be equal.

Nevertheless, the pupils carried out measurements, so they believed that each path is of the same length, which is crucial for connecting the activity with a circle. Even though the paths were not of the same length and did not therefor precisely model a circle, the starting points seemed to be arranged in an arc. This activity is thus potentially inspiring teaching material for the early teaching of circle. Subsequently, the teacher might scaffold pupils to recall this activity when the concept of circle is introduction in a math class. Even more, recalling and reflecting on the activity might be beneficial for grasping the abstract definition of circle.

One more aspect that is worth to be mentioned is the gauge used for determining different paths' lengths. In fact, the finer gauge is used, the more precise the answer is. We, therefor, suggest measuring the distance by using feet instead of steps, as measuring with feet is more precise. Moreover, using feet could be even more beneficial when moving the finish line closer to the start line because the difference between numbers of counted feet would be greater than between numbers of counted steps.

4. Conclusion

Mathematics is primarily an abstract science, but it is generally believed, that the teaching of its concepts should involve as many concrete representations as possible to convey the knowledge to learners. The junior school aged pupils are usually capable of processing the knowledge presented to them on a concrete level of representation. The educators should bear in mind that using concrete objects and real-life examples is beneficial for pupils' knowledge construction. Fortunately, the educational content of school mathematics can be related to a great extent to things and processes in a real-life environment.

In this paper, we proposed a three-phase physical activity as a possible contribution to a constructivist teaching of elementary mathematics. The purpose of this activity was to improve pupils' speed as well as to create a basis for teaching circle. Our overall intention was to assess whether the proposed activity can be applied and whether it is effective in teaching the concept of circle. All three phases of the activity formed an intervention which was carried out at a local primary school with two classes of fourth-graders. The data were collected both during the interventions and immediately after them. The data collection method included participant observation and individual semi-structured interviews. We strived to provide the answer to this research question: "Do pupils connect the segments of the proposed activity with a circle?" The analysis of the obtained data indicates that the pupils were capable of connecting conceptually the designed activity with the concept of circle. Such finding provides arguments for considering the design as a potentially effective tool for teaching the concept of circle, particularly its abstract definition.

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